Globalization, Trade Imbalances and Labor Market Adjustment

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1 Introduction

Suppose the American apparel industry experiences increasing importing competition from China. In response to this shock, employers in apparel are expected to contract or go out of business, leading to displaced workers that need to be reallocated to other industries. In standard models of trade, the surge in imports from China must be accompanied by a surge of exports to the rest of the world, as trade balance is usually imposed. As imports of apparel increase, a trade deficit puts downward pressure on the US dollar. In turn, this currency depreciation propels exports of industries in which the US has a comparative advantage, such as electronics. So, trade-displaced workers in apparel are expected to be reallocated to the electronics industry.

In reality, trade is rarely balanced. Indeed, the large and persistent US trade deficit is considered by some political figures as detrimental to American workers. This project aims to make sense of this sentiment. Revisiting the example from the previous paragraph, suppose that the US can sustain large trade deficits for a prolonged period of time (as is currently the case). In this scenario, import competition in apparel does not need to be met with an expansion of electronics. In turn, the adjustment process behind the reallocation of trade-displaced apparel workers to other sectors of the economy can be more painful and longer than what would be expected in a world where trade balances.

This project explores this idea to make the point that, in a model of trade with endogenous trade deficits and labor market frictions, increasing import competition can lead to a longer adjustment process relative to a model were trade balances period by period. In addition, to the extent that countries such as the US can sustain trade deficits in steady state (Reyes-Heroles (2016)), the effect of trade on unemployment can also be magnified by the presence of endogenous trade deficits.

Therefore, our intuition is that trade deficits can magnify the distributional effects of trade.

We extend the model of trade imbalances in Reyes-Heroles (2016) to allow for labor market frictions. Labor market frictions and dynamics are modeled using elements of Artuç et al. (2010), but we also allow for involuntary unemployment due to search frictions as in Pissarides (2000). We estimate this model using simulated method of moments and data from the World Input-Output Database (WIOD) and the United States Current Population Surveys (CPS). We then use the estimated model to study the effect of different trade shocks on the dynamics of labor market adjustment and on unemployment. To be concrete, we simulate the short to long run effects of (1) productivity shocks in China on adjustment dynamics in the US; (2) a savings glut in China; (3) trade liberalization episodes (unilateral or multilateral); and (4) (potentially asymmetric) changes in trade costs over time.

Our hypotheses are that the adjustment process can be substantially longer in the presence of sustained trade deficits, and steady state effects on unemployment can be larger with sustained deficits. If confirmed, these two hypotheses suggest that trade deficits may have substantial distributional consequences. Moreover, it would imply that the extensive quantitative general equilibrium literature is missing key ingredients.

2 Model

2.1 Environment

There are $i = 1, ..., N$ countries, and $k = 1, ..., K$ sectors. Each country i has a constant labor force given by $\overline{L}_i$ workers/consumers, and within each sector there are a unit of continuum varieties $j \in [0, 1]$. Consumption in each country i aggregates varieties in each sector as follows:

$$C_i^t = \prod_{k=1}^{K} \left(Q_{k,i}^t\right)^{\mu_{k,i}}$$

$$Q_{k,i}^t = \left(\int_0^1 \left(c_{k,i}^t\left(j\right)\right)^{\varepsilon} dj\right)^{\frac{1}{\varepsilon}},$$

where $c_{k,i}^t\left(j\right)$ is the consumption of variety j in sector k and country i at time t, $\mu_{k,i} > 0$, $\sum_k \mu_{k,i} = 1$ $\forall i$, and $\sigma = \frac{1}{1-\varepsilon}$ is the elasticity of substitution across varieties within sectors. P_i^t denotes the associated price index in country i at time t.

2.2 Labor Markets

Workers and firms engage in a costly search process. Firms post vacancies, but not all of them are filled. Workers search for a job, but not all of them are successful. Each variety j constitutes a different labor market. To be specific, the unemployment rate $u_{k,i}^{t}(j)$ is variety-specific, as is the vacancy rate $v_{k,i}^{t}(j)$. Both variables are expressed as a fraction of the labor force $L_{k,i}^{t}(j)$, measured as the sum of workers who are employed or unemployed and searching within sector k / variety j at time t. If $u_{k,i}^{t}(j)$ is the unemployment rate and $v_{k,i}^{t}(j)$ is the vacancy rate, we impose that $m_i\left(u_{k,i}^{t}(j), v_{k,i}^{t}(j)\right)$ matches are formed (as a fraction of the labor force $L_{k,i}^{t}(j)$). The matching function $m_i()$ is increasing in both arguments, concave, and homogeneous of degree 1. Workers face mobility costs across sectors, but there is free mobility across varieties j within a sector. From now on, we drop the index j, but the reader should keep in mind that all labor market variables are country-sector-variety specific.

Define labor market tightness as:

$$\theta_{k,i}^{t} \equiv \frac{v_{k,i}^{t}}{u_{k,i}^{t}}. \tag{1}$$

The probability that any vacancy is matched with an unemployed worker is:

$$\frac{m_i\left(u_{k,i}^{t}, v_{k,i}^{t}\right)}{v_{k,i}^{t}} = m_i\left(\frac{1}{\theta_{k,i}^{t}}, 1\right) \equiv q_i\left(\theta_{k,i}^{t}\right). \tag{2}$$

In turn, the probability that an unemployed worker is matched with an open vacancy is:

$$\frac{m_i\left(u_{k,i}^{t}, v_{k,i}^{t}\right)}{u_{k,i}^{t}} = \frac{v_{k,i}^{t}}{u_{k,i}^{t}} \frac{m_i\left(u_{k,i}^{t}, v_{k,i}^{t}\right)}{v_{k,i}^{t}} = \theta_{k,i}^{t} q_i\left(\theta_{k,i}^{t}\right). \tag{3}$$

In this paper, we adopt the following functional form for the matching function:

$$m_i(u, v) = \frac{uv}{\left(u^{\xi_i} + v^{\xi_i}\right)^{1/\xi_i}}, \tag{4}$$

which is convenient because it guarantees matching rates $q_i\left(\theta_{k,i}^{t}\right)$ and $\theta_{k,i}^{t} q_i\left(\theta_{k,i}^{t}\right)$ that are bounded between 0 and 1.

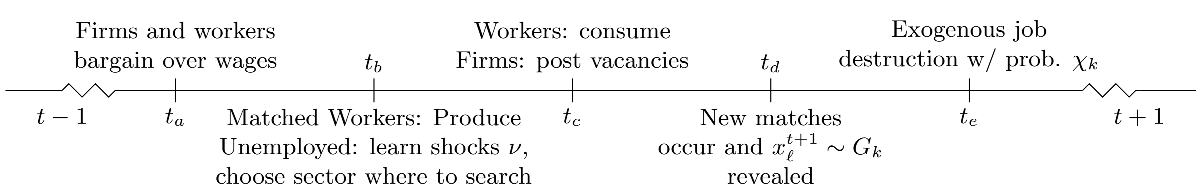
2.3 Households

Countries are organized into representative families, each with a household head that determines consumption, savings, and the allocation of workers across sectors. We first describe the utility of individual workers, then we show how household heads aggregate members' utilities. For ease of notation, we temporarily omit the country subscript i and let ℓ index individuals.

If a worker ended period $t-1$ unemployed in sector k she can either search in sector k at time t (at no additional cost) or incur a moving cost $C_{kk'}$ and search in sector k' at time t—so that $C_{kk} = 0$. If a worker ℓ is not employed at the production stage at t, she receives preference shocks $\left\{\nu_{k,\ell}^t, k = 1, ..., K\right\}$ for each sector at time t. After unemployed workers receive these shocks, the household head decides whether to keep each worker in the same sector and restrict him to search there at t, or to incur a mobility cost and allow him to search in another sector. The $\nu_{k,\ell}^t$ shocks are iid across individuals, sectors and time, and are assumed to follow a Gumbel distribution with parameters $(-\gamma\zeta, \zeta)$ where γ is the Euler-Mascheroni constant and ζ its shape parameter.

After being allocated to search in sector k' at t the unemployed worker receives unemployment utility $b_{k'}$ and matches with a firm with probability $\theta_{k'}^t q(\theta_{k'}^t)$. Once a worker and a firm match at t, a match-specific productivity for $t+1$ production, x_ℓ^{t+1}, is randomly drawn from a distribution $G_{k,i}$, and assumed to be constant over time from then on. At this point, the household head can break a match if keeping it active is not optimal (for example, if the match productivity is too low). Finally, at the end of every period, and following the matching process, there is an exogenous probability χ_k of new and existing matches to dissolve. Successful matches that occur at time t only start to produce at $t+1$, and workers employed in sector k are then paid wages denoted by $w_k^{t+1}\left(x_\ell^{t+1}\right)$. Finally, if a worker produces in sector k she receives a non-pecuniary benefit of η_k. Figure 1 details the timing of the model. Section 2.5 describes the bargaining process that occurs at t_a and section 2.4.2 describes the decision of firms to post vacancies at time t_c.

Figure 1: Timing of the Model



Given this setup, the period utility for individual ℓ at time t is,

$$\mathcal{U}_\ell^t\left(e_\ell^t, k_\ell^{t+1}, k_\ell^t, \nu_\ell^t, c_\ell^t\right) = \left(1 - e_\ell^t\right)\left(-C_{k_\ell^t, k_\ell^{t+1}} + b_{k_\ell^{t+1}} + \nu_{k_\ell^{t+1},\ell}^t\right) + e_\ell^t \eta_{k_\ell^t} + u\left(c_\ell^t\right), \tag{5}$$

where k_ℓ^t is the sector where individual ℓ starts period t (that sector was determined at $t-1$), k_ℓ^{t+1} is the $t+1$ sector of choice (which is decided at time t, interim period t_b in Figure 1), e_ℓ^t is the employment status at the production stage (interim period t_b), $\nu_\ell^t = \left(\nu_{1,\ell}^t, ..., \nu_{K,\ell}^t\right)$, and c_ℓ^t is individual consumption. If individual ℓ is unemployed in sector k at t ($e_\ell^t = 0, k_\ell^t = k$), the individual can switch sectors (from k to k_ℓ^{t+1}), so that mobility costs $C_{k_\ell^t, k_\ell^{t+1}}$, utility of unemployment $b_{k_\ell^{t+1}}$ and shock $\nu_{k_\ell^{t+1},\ell}^t$ are incurred (during period t). On the other hand, if individual ℓ is employed at the production stage at t, $e_\ell^t = 1$, the worker enjoys the non-pecuniary benefit of working in sector k_ℓ^t given by $\eta_{k_\ell^t}$. Finally, individual ℓ also enjoys the utility of consumption $u\left(c_\ell^t\right)$.

Let $\widetilde{e}_k^t\left(x_\ell^{t+1}\right) \in \{0,1\}$ indicate whether the household head continues on with a match at time t given a match productivity of x_ℓ^{t+1} in sector k. We can write the probability that worker ℓ is employed in sector k at time $t+1$, conditional on match productivity x_ℓ^{t+1} and time t information $\left(k_\ell^t, e_\ell^t\right)$ as:

$$\begin{aligned} \Pr\left(k_\ell^{t+1} = k, e_\ell^{t+1} = 1 | x_\ell^{t+1}, k_\ell^t, e_\ell^t\right) = {} & \mathcal{I}\left(k_\ell^t = k\right) e_\ell^t \left(1 - \chi_k\right) \widetilde{e}_k^t\left(x_\ell^{t+1}\right) \\ & + \left(1 - e_\ell^t\right) \mathcal{I}\left(k_\ell^{t+1} = k\right) \theta_k^t q\left(\theta_k^t\right) \left(1 - \chi_k\right) \widetilde{e}_k^t\left(x_\ell^{t+1}\right). \end{aligned} \tag{6}$$

In words, if $\mathcal{I}\left(k_\ell^t = k\right) e_\ell^t = 1$, then worker ℓ is employed in sector k at time t and the match survives with probability $(1 - \chi_k)$ if the family planner decides to keep the match ($\widetilde{e}_k^t\left(x_\ell^{t+1}\right) = 1$). If $e_\ell^t = 0$, that is, the worker is unemployed at t, and the planner chooses $k_\ell^{t+1} = k$, then the worker is employed in sector k at time $t+1$ with probability $\theta_k^t q\left(\theta_k^t\right)(1 - \chi_k)\widetilde{e}_k^t\left(x_\ell^{t+1}\right)$. Importantly, workers' sector and employment status at $t+1$, k_ℓ^{t+1} and e_ℓ^{t+1}, are determined by actions taken at t.

The head of the household aggregates (5) over family members and maximizes the net present value of utility subject to her budget constraint, and the controlled process on employment (6). In addition to consumption and employment decisions, the household head has access to international financial markets by means of buying and selling one-period riskless bonds that are available in zero-net supply around the world. One can think of international bond markets in period t as spot markets in which a family buys a piece of paper with face value of B^{t+1} in exchange for a bundle of goods with the same value, and the piece of paper represents a promise to receive goods in period $t+1$ with a value equal to $R^{t+1}B^{t+1}$. Nominal returns R^{t+1} are equalized across countries. Finally, the household collects and aggregates profits across all firms, Π^t, but takes this lump sum payment as given. The household head chooses the path of consumption, c_ℓ^t, the path of sectoral choices, k_ℓ^t, employment decisions, $\widetilde{e}_k^t(x)$, and bonds, B^t, to solve:

$$\max_{\left\{k_{\ell,}^t, \widetilde{e}_k^t(.), B^t, c_\ell^t\right\}} E_0 \left\{ \sum_{t=0}^{\infty} \delta^t \int_0^{\overline{L}} \mathcal{U}_\ell^t d\ell \right\} \tag{7}$$

Subject to (6) and the budget constraint:

$$P^t \int_0^{\overline{L}} c_\ell^t d\ell + B^{t+1} - \Pi^t - R^t B^t - \int_0^{\overline{L}} \left(\sum_{k=1}^{K} \mathcal{I}\left(k_\ell^t = k\right) e_\ell^t w_k^t \left(x_\ell^t\right) \right) d\ell \leq 0. \tag{8}$$

The budget constraint states the family can buy consumption or bonds for next period using profits and wage income, net of interest payments (or collections) on past bonds. Let $\widetilde{\lambda}^t$ be the Lagrange multiplier on the family's budget constraint (8). The optimality condition with respect to c_ℓ^t is $u'\left(c_\ell^t\right) = \widetilde{\lambda}^t P^t$, so that individual consumption is equalized across individuals within the household: $c_\ell^t = c^t \ \forall \ell$. Henceforth, we will refer to c^t as per capital consumption. Armed with this observation, Appendix A shows that (7) subject to (8) and (6) can be rewritten recursively for unemployed and employed workers. We now turn this recursive formulation.

Return to indexing countries by i. Since workers are symmetric up to x and η in each country, we stop indexing individual workers. We denote by $\widetilde{U}_{k,i}^t(\nu^t)$ the value of unemployment in sector k, country i at time t conditional on shocks ν^t, and by $W_{k,i}^t\left(x\right)$ the value of employment conditional on match-specific productivity x. The sector decision policy and the continuation rule $\widetilde{e}_k^t\left(.\right)$ must solve, conditional on shocks ν^t:

$$\widetilde{U}_{k,i}^t(\nu^t) = \max_{k'} \left(\begin{array}{c} -C_{kk',i} + \nu_{k',\ell}^t + b_{k',i} \\ + \left(1 - \chi_{k',i}\right) \theta_{k',i}^t q\left(\theta_{k',i}^t\right) \delta \int_{x_{\min}}^{x_{\max}} \max\left\{ W_{k',i}^{t+1}\left(x\right), U_{k',i}^{t+1} \right\} dG_{k',i}\left(x\right) \\ + \left(1 - \left(1 - \chi_{k',i}\right) \theta_{k',i}^t q\left(\theta_{k',i}^t\right)\right) \delta U_{k',i}^{t+1}, \end{array} \right), \tag{9}$$

and

$$W_{k,i}^t\left(x\right) = \widetilde{\lambda}_i^t w_{k,i}^t\left(x\right) + \eta_{k,i} + \delta\left(1 - \chi_{k,i}\right) \left(\max\left\{ W_{k,i}^{t+1}\left(x\right), U_{k,i}^{t+1} \right\} \right) + \delta \chi_{k,i} U_{k,i}^{t+1}. \tag{10}$$

In equation (9), $U_{k,i}^t \equiv E_\nu \left(\widetilde{U}_{k,i}^t(\nu^t) \right)$ is the expected value of $\widetilde{U}_{k,i}^t(\nu^t)$ integrated over ν^t. The first line in equation (9) corresponds to the costs of switching sectors, $-C_{kk',i} + \nu_{k',i}^t$, as well as the sector-specific value of being unemployed in that sector $b_{k'i}$. The second line is the probability of a match $\left(1 - \chi_{k',i}\right) \theta_{k',i}^t q\left(\theta_{k',i}^t\right)$ that is not exogenously destroyed multiplied by the discounted value of the match – notice that for low values of $W_{k',i}^{t+1}\left(x\right)$, the household head dissolves the match so that the worker obtains $U_{k',i}^{t+1}$. Finally, the third line is the discounted value of being unemployed next period if the worker fails to successfully match.

In equation (10), the first term on the left hand side is the wage function, multiplied by the household head's Lagrange multiplier on the budget constraint $\widetilde{\lambda}_i^t$. The second term is the non-pecuniary benefit of working in sector k in country i. The next terms are the continuation values: with probability $(1-\chi_{k,i})$ the match does not exogenously dissolve and the worker continues the match; with probability $\chi_{k,i}$ the match exogenously breaks and the worker receives the value of unemployment in k.

Since ν is Gumbel distributed, the policy rule for unemployed workers can be solved analytically. Writing the transition probabilities is simpler if one rearranges (9) to:

$$\widetilde{U}_{k,i}^t(\nu^t) = \max_{k'} \left\{ \begin{array}{c} -C_{kk',i} + \nu_{k',i}^t + b_{k',i} + \\ (1-\chi_{k',i})\theta_{k',i}^t q(\theta_{k',i}^t)\delta \int_{x_{\min}}^{x_{\max}} \max\left\{ W_{k',i}^{t+1}(x) - U_{k',i}^{t+1}, 0 \right\} dG_{k',i}(x) + \delta U_{k',i}^{t+1} \end{array} \right\} \tag{11}$$

With this notation, the transition rates take the logit form:

$$s_{kk',i}^{t+1} = \frac{\exp\left\{ \begin{array}{c} -C_{kk',i} + b_{k',i} + \\ (1-\chi_{k',i})\theta_{k',i}^t q(\theta_{k',i}^t)\delta \int_{x_{\min}}^{x_{\max}} \max\left\{ W_{k',i}^{t+1}(x) - U_{k',i}^{t+1}, 0 \right\} dG_{k',i}(x) + \delta U_{k',i}^{t+1} \end{array} \right\}}{\sum_{k''} \exp\left\{ \begin{array}{c} -C_{kk''i} + b_{k''i} + \\ (1-\chi_{k'',i})\theta_{k'',i}^t q(\theta_{k'',i}^t)\delta \int_{x_{\min}}^{x_{\max}} \max\left\{ W_{k'',i}^{t+1}(x) - U_{k'',i}^{t+1}, 0 \right\} dG_{k'',i}(x) + \delta U_{k'',i}^{t+1} \end{array} \right\}}. \tag{12}$$

2.4 Firms

2.4.1 Incumbents

Firms are price takers and their revenue is given by $p_{k,i}^t(j) z_{k,i}(j) x$, where $p_{k,i}^t(j)$ is the price of variety j in sector k in country i at time t, and $z_{k,i}(j)$ is a common productivity term available to all firms producing variety j operating in i. We assume that conditional on entry, changing varieties is costless. Hence, competition across varieties will ensure that $p_{k,i}^t(j) z_{k,i}(j) = p_{k,i}^t(j') z_{k,i}(j')\ \forall j, j'$. Hence, wages and profits only depend on x, but not j. Because of this, we again omit j until discussing trade. When a firm with productivity x produces, it pays a wage $w_{k,i}^t(x)$ to its employee. We can write the value function for incumbent workers, $J_{k,i}^t(x)$, as:

$$J_{k,i}^t(x) = \widetilde{\lambda}_i^t \left(p_{k,i}^t z_{k,i} x - w_{k,i}^t(x) \right) + (1-\chi_{k,i})\,\delta \max\left\{ J_{k,i}^{t+1}(x), 0 \right\}. \tag{13}$$

The first term is the firm's current profit. Notice that the firm values profits with the same multiplier, $\widetilde{\lambda}_i^t$, as households. Through the Euler equation that we derive in Section 2.8, this is equivalent to firms discounting future profits at $(R^t)^{-1}$. However, this notation is useful as it keeps the units of the value functions between unemployed workers, employed workers, and firms, consistent.

If $J_{k,i}^{t}(x) < 0$ the firm does not produce and exits. To simplify the exposition, we anticipate that $J_{k,i}^{t}(x)$ is an increasing function of x, so that the decision to exit takes the form of a cutoff rule. Define $\underline{x}_{k,i}^{t}$ as the value solving $J_{k,i}^{t}\left(\underline{x}_{k,i}^{t}\right) = 0$.

2.4.2 New Entrants

Potential entrants can match by posting vacancies in k (and variety j), at a cost of $\kappa_{k,i} p_{k,i}^{t} z_{k,i}$. Vacancies are posted at the interim period t_c as illustrated in Figure 1. As argued above, the ability to switch varieties equates $p_{k,i}^{t} z_{k,i}$ across varieties, hence this distinction can be ignored. If a firm successfully matches with a worker at t, production starts at $t+1$. If we denote the expected value of an open vacancy by $V_{k,i}^{t}$, then:

$$V_{k,i}^{t} = -\widetilde{\lambda}_{i}^{t} \kappa_{k,i} p_{k,i}^{t} z_{k,i} + \delta \left[\begin{array}{c} q_i\left(\theta_{k,i}^{t}\right)(1-\chi_{k,i}) \int_{\underline{x}_{k,i}^{t+1}}^{x_{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s) \\ +\left(1 - q_i\left(\theta_{k,i}^{t}\right)(1-\chi_{k,i})\right) \max\left\{V_{k,i}^{t+1}, 0\right\} \end{array} \right]. \tag{14}$$

The first term on the right hand side is the cost of posting vacancies. The parameter $\kappa_{k,i}$ governs the scale of these costs. The second term says that in the next period entrants match with probability $q_i\left(\theta_{k,i}^{t}\right)$ and obtain the expected value of $J_{k,i}^{t+1}$ starting in the next period, if the match is not exogenously destroyed. If they do not match (or the match is destroyed), they can post another vacancy.

There is an infinite mass of potential entrants, and there is free entry so that $V_{k,i}^{t} \leq 0 \ \forall k, i, t$. Throughout the paper, we impose that this holds with equality in equilibrium—both in steady state and in any transitions.

2.5 Wages

After meeting, workers and firms bargain over their match surplus every period to decide on wages. Although matching occurs at $t-1$ (interim period t_c), bargaining over wages occurs always at the beginning of period t, and, more specifically, during interim period t_a—see Figure 1. We assume Nash bargaining, so that, if $\beta_{k,i}$ is the worker's bargaining power and the firm's outside option is to shutdown, the wage $w_{k,i}^{t}(x)$ solves:

$$W_{k,i}^{t}(x) - U_{k,i}^{t} = \beta_{k,i}\left(J_{k,i}^{t}(x) + W_{k,i}^{t}(x) - U_{k,i}^{t}\right). \tag{15}$$

Notice that, according to (15), $W_{k,i}^t(x) \geq U_{k,i}^t$ whenever $J_{k,i}^t(x) \geq 0$. The solution to the bargaining problem leads to the wage schedule below:

$$w_{k,i}^t(x) = \beta_{k,i} p_{k,i}^t z_{k,i} x + (1-\beta_{k,i}) \frac{\left(U_{k,i}^t - \delta U_{k,i}^{t+1} - \eta_{k,i}\right)}{\widetilde{\lambda}_i^t}. \tag{16}$$

This is similar to the standard wage equation in search models: the worker's wage is a weighted average of output, and a function of their outside option. The only new term is the non-pecuniary benefit, which is subtracted from the outside option. By integrating wages across all individuals in the economy, one can solve for the family's total wage income.

Plugging (16) into (13), we obtain a new expression for $J_{k,i}^t(x)$, confirming that it is increasing in x.

$$\begin{aligned} J_{k,i}^t(x) = {} & (1-\beta_{k,i}) \widetilde{\lambda}_i^t p_{k,i}^t z_{k,i} x - (1-\beta_{k,i}) \left(U_{k,i}^t - \delta U_{k,i}^{t+1} - \eta_{k,i}\right) \\ & + (1-\chi_{k,i}) \delta \max\left\{J_{k,i}^{t+1}(x), 0\right\}. \end{aligned} \tag{17}$$

2.6 Labor Market Dynamics

Period t starts with sector-specific unemployment rate $\widetilde{u}_{k,i}^{t-1}$ and labor force $L_{k,i}^{t-1}$. $u_{k,i}^t$ is the share of sector-k workers searching for a job (in sector k) at t (measured just before t_d, but after unemployed workers choose their sectors). It is given by:

$$u_{k,i}^t = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t-1} \widetilde{u}_{\ell,i}^{t-1} s_{\ell k,i}^t}{L_{k,i}^t}, \tag{18}$$

where $L_{k,i}^t$ is the number of employed or unemployed workers searching in sector k at t (more precisely at t_c) and is equal to:

$$L_{k,i}^t = L_{k,i}^{t-1} + IF_{k,i}^t - OF_{k,i}^t, \tag{19}$$

with

$$IF_{k,i}^t \equiv \sum_{\ell \neq k} L_{\ell,i}^{t-1} \widetilde{u}_{\ell,i}^{t-1} s_{\ell k,i}^t, \tag{20}$$

and

$$OF_{k,i}^t \equiv L_{k,i}^{t-1} \widetilde{u}_{k,i}^{t-1} \left(1 - s_{kk,i}^t\right). \tag{21}$$

Only firms with $x \geq \underline{x}_{k,i}^{t+1}$ produce at $t+1$, but it is important to keep in mind that the productivity threshold $\underline{x}_{k,i}^{t+1}$ is actually determined at time t. Perfect foresight is an important assumption, as the decision to remain in the market or exit occurs before prices are formed. The *number* of jobs created in sector k is given by:

$$JC_{k,i}^{t} = L_{k,i}^{t} u_{k,i}^{t} \theta_{k,i}^{t} q_i \left(\theta_{k,i}^{t}\right) \left(1 - \chi_{k,i}\right) \left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right), \tag{22}$$

and the number of jobs destroyed is given by:

$$\begin{aligned} JD_{k,i}^{t} &\equiv \left(\chi_{k,i} + \left(1 - \chi_{k,i}\right) \Pr\left(\underline{x}_{k,i}^{t} \leq x < \underline{x}_{k,i}^{t+1} | \underline{x}_{k,i}^{t} \leq x\right)\right) L_{k,i}^{t-1} \left(1 - \widetilde{u}_{k,i}^{t-1}\right) \\ &= \left(\chi_{k,i} + \left(1 - \chi_{k,i}\right) \max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}, 0\right\}\right) L_{k,i}^{t-1} \left(1 - \widetilde{u}_{k,i}^{t-1}\right), \end{aligned} \tag{23}$$

where $\Pr\left(\underline{x}_{k,i}^{t} \leq x < \underline{x}_{k,i}^{t+1} | \underline{x}_{k,i}^{t} \leq x\right)$ is the share of active firms above the productivity threshold at t but below at $t+1$ (endogenous exit). Therefore, the rate of unemployment at the end of period t, after job creation and job destruction, is given by:

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t} u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}. \tag{24}$$

This technically takes place at the variety level and needs to be added up in equilibrium. However, as we'll argue flesh out in discussing goods markets, the fact that $p_{k,i}^{t} z_{k,i}$ is equalized across varieties implies all outcomes across varieties are symmetric.

Equations (18)-(24) describe the evolution of labor market stocks over time. In any given period, these stocks are bound by the labor market clearing condition:

$$\sum_{k=1}^{K} L_{k,i}^{t} = \overline{L}_i. \tag{25}$$

2.7 Goods Market and International Trade

All goods are tradable across countries. Trade occurs at the variety level, so we return to indexing output by j. Each variety j from sector k produced in i can be purchased domestically at price $p_{k,i}^{t}(j)$. However, consumers in i also have the option of purchasing variety j from country o at price $d_{k,oi}^{t} p_{k,o}^{t}(j)$, where $p_{k,o}^{t}(j)$ is the price of variety j from sector k in country o and $d_{k,oi}^{t} > 1$ is an iceberg trade cost of transporting the variety from exporter o to importer i. Consumers shop around the world for the lowest possible price for each variety. Therefore, consumersin country i pay

$$\min_{o\in\{1,\ldots,N\}} \left\{ d_{k,oi}^{t} p_{k,o}^{t}\left(j\right) \right\}. \tag{26}$$

In any country i, the productivity component $z_{k,i}(j)$ is drawn from a Frechet distribution

$$F_{k,i}\left(z\right) \exp\left(-\left(A_{k,i}\right)^{\lambda} z^{-\lambda}\right). \tag{27}$$

The parameter $A_{k,i} > 0$ is related to the location of the distribution: a larger $A_{k,i}$ implies larger $z_{k,i}$ draws on average. $\lambda > 1$ governs the dispersion of the $z_{k,i}$ draws. A lower λ leads to more dispersion in $z_{k,i}$ draws. Note that $p_{k,i}^{t}\left(j\right) z_{k,i}\left(j\right) = p_{k,i}^{t}\left(j'\right) z_{k,i}\left(j'\right) \, \forall j, j'$. We will define the sectoral price, $\widetilde{w}_{k,i}^{t}$ to be the equilibrium value of $p_{k,i}^{t}\left(j\right) z_{k,i}\left(j\right)$ in each country, sector and period. Given this notation, we have the following expression for the price of individual varieties:

$$p_{k,i}^{t}\left(j\right) = \frac{\widetilde{w}_{k,i}^{t}}{z_{k,i}\left(j\right)}, \tag{28}$$

Our sectoral prices, $\tilde{w}_{k,i}^{t}$ are a clear analog to unit costs in Eaton and Kortum (2002). Given iceberg trade costs, prices of goods shipped from an exporter o to an importer i are draws from the random variable

$$P_{k,oi}^{t} = \frac{d_{k,oi}^{t} \widetilde{w}_{k,o}^{t}}{Z_{k,o}}, \tag{29}$$

With *cdf*

$$\Pr\left(P_{k,oi}^{t} \leq p\right) = 1 - \exp\left(-\left(\frac{pA_{k,o}}{d_{k,oi}\widetilde{w}_{k,o}^{t}}\right)^{\lambda}\right) \tag{30}$$

Since consumers search for the lowest price, the distribution of prices actually paid by country i is given by:

$$H_{k,i}^{t}\left(p\right) = 1 - \prod_{o'=1}^{N} \left(1 - \Pr\left(P_{k,oi}^{t} \leq p\right)\right) = 1 - \exp\left(-\Phi_{k,i}^{t} p^{\lambda}\right), \tag{31}$$

where

$$\Phi_{k,i}^{t} = \sum_{o'} \left(\frac{A_{k,o'}}{d_{k,o'i}^{t} \widetilde{w}_{k,o'}^{t}}\right)^{\lambda}. \tag{32}$$

The exact price index is given by

$$P_{i}^{t} = \prod_{k=1}^{K} \left(\frac{P_{k,i}^{t}}{\mu_{k,i}}\right)^{\mu_{k,i}}, \tag{33}$$

where

$$P_{k,i}^{t} = \psi \left(\Phi_{k,i}^{t}\right)^{-\frac{1}{\lambda}}, \tag{34}$$

$$\psi = \left[\Gamma\left(\frac{\lambda+1-\sigma}{\lambda}\right)\right]^{\frac{1}{1-\sigma}}. \tag{35}$$

As in Eaton and Kortum (2002), consumers in country i will spend a share $\pi_{k,oi}^{t}$ of their sector-k expenditures on goods from country o given by:

$$\frac{E_{k,oi}^{t}}{E_{k,i}^{t}} = \pi_{k,oi}^{t} = \frac{\left(\frac{A_{k,o}}{d_{k,oi}^{t}\widetilde{w}_{k,o}^{t}}\right)^{\lambda}}{\Phi_{k,i}^{t}}, \tag{36}$$

where $E_{k,i}^{t}$ is total expenditure of country i on sector k goods and $E_{k,oi}^{t}$ is total expenditure of country i on sector k goods produced by country o:

$$E_{k,i}^{t} = \sum_{o=1}^{N} E_{k,oi}^{t}. \tag{37}$$

Total revenue from sector k in country o is given by:

$$Y_{k,o}^{t} = \widetilde{w}_{k,o}^{t} L_{k,o}^{t-1}\left(1-\widetilde{u}_{k,o}^{t-1}\right)\int_{\underline{x}_{k,o}^{t}}^{x_{\max}} \frac{s}{1-G_{k,o}\left(\underline{x}_{k,o}^{t}\right)} dG_{k,o}\left(s\right). \tag{38}$$

Market clearing dictates that:

$$Y_{k,o}^{t} = \sum_{i=1}^{N} \pi_{k,oi}^{t}\mu_{k,i}E_{i}^{t} + \kappa_{k,o}\widetilde{w}_{k,o}^{t}\theta_{k,o}^{t}u_{k,o}^{t}L_{k,o}^{t}, \tag{39}$$

where $E_{i}^{t} = \sum_{k=1}^{K} E_{k,i}^{t}$ is total expenditure in country i, $\sum_{i=1}^{N} \pi_{k,oi}^{t}\mu_{k,i}E_{i}^{t}$ is total expenditure on final goods produced in country o, and $\kappa_{k,o}\widetilde{w}_{k,o}^{t}\theta_{k,o}^{t}u_{k,o}^{t}L_{k,o}^{t}$ is total expenditure on vacancy costs in sector k, country o at time t. These costs are incurred in terms of sector k goods produced by country o. We will denote by $VA_{k,o}^{t}$ the value added in sector k, country o at time t:

$$VA_{k,o}^{t} \equiv Y_{k,o}^{t} - \kappa_{k,o}\widetilde{w}_{k,o}^{t}\theta_{k,o}^{t}u_{k,o}^{t}L_{k,o}^{t}.$$

The gap between country i's total value added and total expenditure is a trade imbalance and is defined as:

$$NX_i^t \equiv \sum_{k=1}^{K} VA_{k,i}^t - E_i^t. \tag{40}$$

We now model how NX_i^t is determined in equilibrium.

2.8 Trade Imbalances

The household's budget constraint (8) at t can be rewritten as:

$$P_i^t C_i^t + B_i^{t+1} = \sum_{k=1}^{K} VA_{k,i}^t + R^t B_i^t, \tag{41}$$

where $C_i^t = \overline{L}_i c_i^t$ is aggregate consumption.

Bonds are in zero net supply, $\sum_i B_i^t = 0$, and initial conditions are given by $W_i^0 \equiv R^0 B_i^0$ with $\sum_{i=1}^{N} W_i^0 = 0$. The solution to the household head's problem (7) must satisfy the following Euler Equation:

$$\frac{u'(c_i^t)/P_i^t}{u'(c_i^{t+1})/P_i^{t+1}} = \delta R^t. \tag{42}$$

If we assume log utility, $u(c) = \ln(c)$, then we can solve for the equilibrium interest rate recursively in closed form. Notice that the Euler equations become

$$P_i^{t+1} C_i^{t+1} = \delta R^{t+1} P_i^t C_i^t \ , \tag{43}$$

which imply that

$$\sum_{i=1}^{N} P_i^{t+1} C_i^{t+1} = \delta R^{t+1} \sum_{i=1}^{N} P_i^t C_i^t. \tag{44}$$

Imposing for every period that:

$$\sum_{i=1}^{N} B_i^{t+1} = 0, \tag{45}$$

and using the family's budget constraint (8), we obtain:

$$\sum_{i=1}^{N} \left(P_i^t C_i^t + B_i^{t+1} \right) = \sum_{i=1}^{N} \left(VA_i^t + R^t B_i^t \right) \Rightarrow \sum_{i=1}^{N} P_i^t C_i^t = \sum_{i=1}^{N} VA_i^t, \tag{46}$$

where $VA_i^t = \sum_{k=1}^{K} VA_{k,i}^t$ is total country i value added. Therefore, bond returns are given by:

$$R^{t+1} = \frac{1}{\delta} \frac{\sum_{i=1}^{N} VA_i^{t+1}}{\sum_{i=1}^{N} VA_i^t}. \tag{47}$$

Finally, we need a condition linking the financial and goods markets in each country:

$$NX_i^t \equiv \sum_{k=1}^{K} \left(VA_{k,i}^t - E_{k,i}^t \right) = B_i^{t+1} - R^t B_i^t. \tag{48}$$

2.9 Equilibrium

An equilibrium in this model is a set of initial allocations, $\{L_{k,i}^0, \underline{x}_{k,i}^0, B_i^0,\}$, a final steady state, $\{L_{k,i}^\infty, \underline{x}_{k,i}^\infty, B_i^\infty,\}$ and sequences of policy functions for workers/firms, $\{s_{kk',i}^t, \underline{x}_{k,i}^t, w_{k,i}^t(x)\}$, value functions for workers/firms, $\{U_{k,i}^t, W_{k,i}^t, J_{k,i}^t\}$, labor market tightnesses, $\{\theta_{k,i}^t\}$, bond decisions by the households, B_i^t, allocations, $\{L_{k,i}^t, u_{k,i}^t, c_{k,i}^t\}$, profits and household expenditure, $\{\Pi_i^t, E_i^t\}$, trade shares, $\left\{\pi_{k,io}^t\right\}$, and sectoral prices, $\{\widetilde{w}_{k,i}^t\}$ such that:

1. Worker and firms' value functions solve (9), (10), and (13).
2. Prices and labor market tightness ensure the free entry condition holds in each country and sector:
$$V_{k,i}^t = 0 \ \forall k, i, t$$
3. The wage equation solves the Nash bargaining problem and is given by (16).
4. Allocations and unemployment rates evolve accoridng to (18), (19), (24).
5. Consumption and bonds decisions solve the household's dynamic consumption-savings problem (7)-(8) + (6).
6. Goods markets clear: equation (39) is met.
7. Labor markets clear: $\sum_{k=1}^{K} L_{k,i}^t = \overline{L}_i$.
8. Bonds markets clear: $\sum_{i=1}^{N} B_i^t = 0$.
9. The final steady state is such that allocations are consistent with the invariant distribution of $s_{kk',i}^\infty$.

3 Data

To estimate the model, we use data on trade shares, employment allocations, average wages, value added, share of value added going to labor payments (the labor share), and unemployment rates from the World Input Output Database (WIOD). To be able to identify mobility costs, we use data on worker flows across industries as well as into and out of non-employment. Flows of workers are most readily available in the US, and we use the Current Population Survey (CPS) to compute these statistics. Finally, to be able to pin down the distribution of match productivities $G_{k,i}$, we compute the dispersion of wages in the United States using the CPS. Because we cannot consistently measure inter-sectoral labor flows and wage dispersion across countries, we impose common inter-sectoral mobility costs and match productivity dispersion across countries.

In order to make the dimension of the model and estimation procedure tractable, we divide the economy into 5 sectors and 6 countries. Tables 1 and 2 detail these divisions.

Table 1: Sector Definitions

1	Low-Tech Manufacturing	Wood products; Paper, printing and publishing; Coke and refined petroleum; Basic and fabricated metals; Other manufacturing
2	Mid-Tech Manufacturing	Food, beverage and tobacco; Textiles; Leather and footwear; Rubber and plastics; Non-metallic mineral products.
3	High-Tech Manufacturing	Chemical products; Machinery; Electrical and optical equipment; Transport equipment.
4	Energy and Others	Energy, Mining and quarrying; Agriculture, Forestry and fishing;
5	Services	Utilities; Construction; Sale, maintenance and repair of motor vehicles and motorcycles; Retail sale of fuel; Wholesale trade; Retail trade; Hotels and restaurants; Land transport; Water transport; Air transport; Other transport services; Post and telecommunications; Financial, real estate and business services; Government, education, health and other services; Households with employed persons.

Table 2: Country Definitions

1	China
2	European Union
3	United Kingdom
4	Rest of the World (low income)
5	Rest of the World (high income)
6	United States

Notes: Rest of the World (high income) = {Australia, Japan, Canada, South Korea, Taiwan} and Rest of the World (low income) = {Brazil, Indonesia, India, Mexico, Turkey and Russia}

4 Estimation

To complete the specification of the model, we assume that the distribution of match productivities $G_{k,i} = \log \mathcal{N}(0, (\sigma^x_{k,i})^2)$, and assume that worker have log utility. Table 3 summarizes the parameters of the model that are needed for the simulation of counterfactuals. These parameters are either imposed to typical values used in the literature, calibrated without having to solve the full model, or jointly estimated using the method of simulated moments (MSM). Among the parameters fixed at values commonly used in the literature, we follow Artuç et al. (2010) and impose that the dispersion of the ν shocks is driven by the scale parameter $\zeta_i = 1.63\ \forall i$, and the discount factor to be 0.97 per year. One period in our model corresponds to a quarter in the data, so that we will be imposing $\delta = 0.97^{1/4}$. We fix the Frechet scale parameter at $\lambda = 4$, as estimated by Simonovska and Waugh (2014) . Finally, we use the value of $\xi_i = 1.84\ \forall i$ estimated by Coşar et al. (2016).

Among the parameters calibrated without having to solve the model, trade costs $d_{k,oi}$ are obtained assuming symmetric trade costs and the Head-Ries index (Head and Ries, 2001)

$$\widehat{d}^t_{k,oi} = \frac{\pi^t_{k,io}}{\pi^t_{k,oo}} \frac{\pi^t_{k,oi}}{\pi^t_{k,ii}}, \tag{49}$$

and $\mu_{k,i}$ can be directly measured using WIOD data on sector and country-specific expenditures.

The remaining parameters are estimated using the method of simulated moments to match trade shares, employment allocations, average wages, value added, labor shares, national unemployment rates, and 1-year transition rates between sectors. In particular, we assume that the world is in steady state at the year 2000. Given a guess of parameters we solve for the steady state and compute moments that we can compare to the data. Given the high dimensionality of the parameter space,

Table 3: Summary of Parameters

Parameter	Description	Source
δ	Discount factor	Artuç et al. (2010)
ζ_i	Dispersion of ν shocks	Artuç et al. (2010)
ξ_i	Matching Function	Coşar et al. (2016)
λ	Frechet Scale Parameter	Simonovska and Waugh (2014)
$d_{k,oi}$	Trade Costs	Head-Ries index
$\mu_{k,i}$	Expenditure Shares	WIOD
$A_{k,o}$	Frechet Location Parameters	MSM
$\kappa_{k,i}$	Vacancy Costs	MSM
$\chi_{k,i}$	Exogenous Exit	MSM
$\sigma^2_{k,i}$	$G_{k,i}$	MSM
$C_{kk'}$	Mobility Costs	MSM
$b_{k,i}$	Unemployment Utility	MSM
$\beta_{k,i}$	Worker Bargaining Power	MSM
$\sigma_{k,i}$	Dispersion of Match Productivities	MSM

we impose restrictions to reduce the number of parameters to be estimated. We outline the number of parameters to be estimated via MSM below, and describe what restrictions we impose.

- $A_{k,o}$: $K \times N - 1$ parameters (one of the productivity parameters needs to be normalized)
- $b_{k,i} = b_i$: N parameters
- $\beta_{k,i}$: $K \times N$ parameters
- $\sigma^x_{k,i} = \sigma^x$: 1 parameter
- $\kappa_{k,i} = \kappa_i$: N parameters
- $\chi_{k,i} = \chi_i$: N parameters
- $\eta_{k,i}$: $(K-1) \times N$ parameters
- $C_{kk',i} = \frac{P_i}{P_{US}} C_{kk',US}$: $K \times (K-1)$ parameters

Table 4 summarizes the data moments our method of simulated moments targets. Appendix C.1 details how we generate the model counterparts of these.

Table 4: Summary of Targeted Moments

Targeted Moment	Source
Employment allocations across sectors and countries	WIOD
Average wages across sectors and countries	WIOD
National unemployment rates	WIOD
Value added across sectors and countries	WIOD
Labor Shares	WIOD
Trade shares	WIOD
Net exports	WIOD
Coefficient of variation of log-wages in the United States	CPS
Yearly transition rates for the United States	CPS

5 Results

5.1 Model Fit

Given the highly non-linear (and non-concave) objective function, and the high dimension of the parameter space, estimation is challenging. We are still working on improving our estimates, but our preliminary results are encouraging, leading to a good overall fit of the data. For instance the average deviation from data moments to model generated moments is of 25%. Figure 2 plots all the moments we target in the data vs model generated moments.

Although most moments are very well matched, our model should do a better job in a few obvious dimensions. For instance, our model underestimates yearly transitions from unemployment to unemployment by a factor of nearly 5 (these transitions in the CPS data amount to 30%, whereas our model predicts 6%). Although we match unemployment rates reasonably well across countries, we are still overestimating the unemployment rate in the United States (5.6% in the model vs. 2.9% in the data), and underestimating it for the Rest of the World (developed) (1.2% in the model vs. 3.6% in the data). Otherwise, average wages, employment shares, value added and trade shares are very well matched. Figures 3a to 4c show how each set of moments is matched by the model.

Figure 2: All Data Moments plotted against All Model Generated Moments

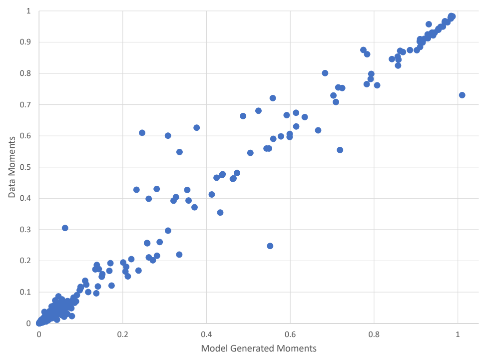
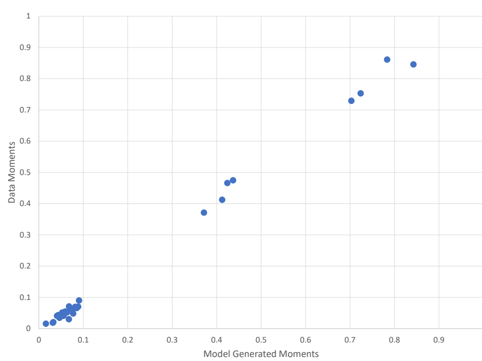
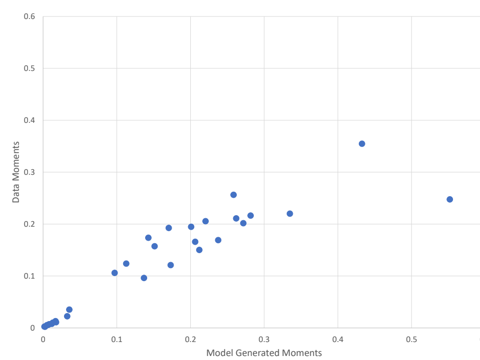


Figure 3: Data Moments vs Model Generated Moments

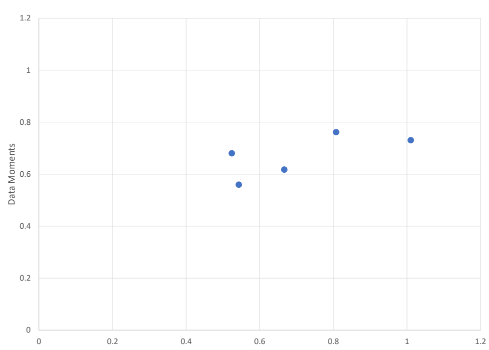
(a) Employment Shares



(b) Average Wages



(c) Coefficient of Variation on Log Wages



(d) Unemployment Rates

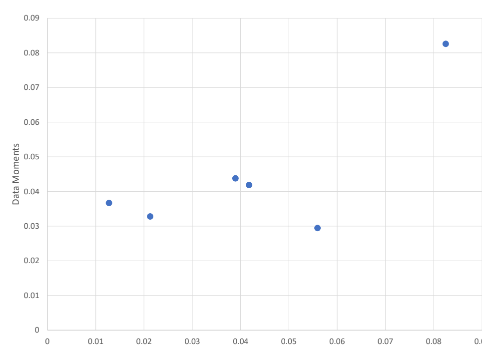
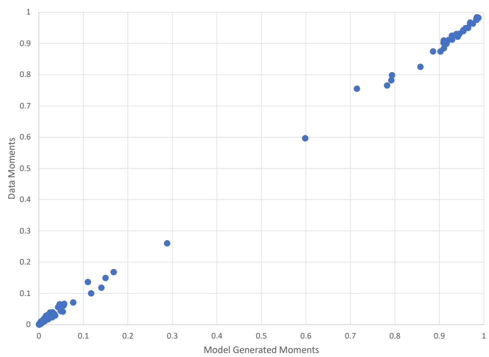
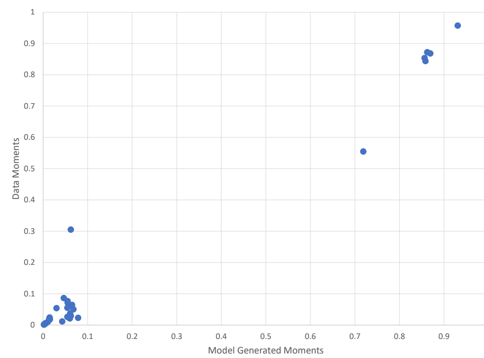


Figure 4: Data Moments vs Model Generated Moments, Cont'd.

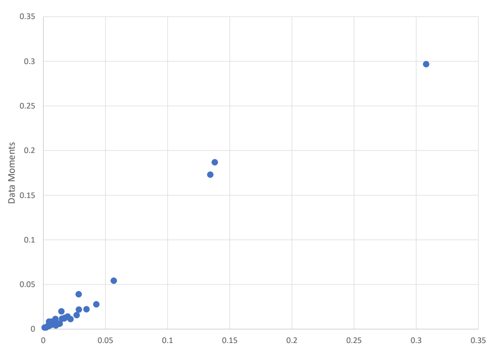
(a) Trade Shares



(b) US Transition Rates



(c) Value Added



5.2 Counterfactuals

To analyze the mechanisms in our model, we consider an across-the-board doubling of Chinese productivity. That is to say, we consider a sudden and unanticipated shock at time 0 where $A^t_{CN,k} = 2 \times A^{-1}_{CN,k}$ for all sectors and for all $t \geq 0$. This is obviously a large shock to occur in a single period, but the size helps magnify the model's mechanisms. Table 5 displays some proportional changes, x^{∞}/x^{-1}, (noted by $\hat{\Delta}$) for several key outcomes across steady states.

Table 5: Steady State Outcomes

Country	$\hat{\Delta}$ Real Income	$\hat{\Delta}$ Unemployment	$\hat{\Delta}$ Exports/GDP	$\hat{\Delta}$ Exports/World GDP
China	1.992	0.999	0.857	1.504
EU	0.967	0.994	1.466	1.334
UK	1.011	0.989	1.041	0.989
ROW (L)	0.985	1.015	1.419	1.328
ROW (H)	0.992	1.010	1.176	1.137
USA	1.016	0.986	0.684	0.696

The first thing to note is that real income nearly doubles in China, but changes are quite small across other countries. That Chinese income doubles due to a doubling of productivity is no surprise, while the relatively muted effects across other countries are in line with small gains from trade in models like ours. It is worth noting that a doubling of Chinese productivity is still a small change. For example, the Chinese world GDP increases from 3.6% to 6.3%. In the data, China's share of global production actually increases from 3.6% to over 15%. The negative impacts on some countries likely reflect changes in the prices of those countries' comparative advantage goods. As an example, the estimated sectoral productivities of China are highly correlated with both ROW combinations, but less so with the US (and negatively correlated with the UK). Unemployment rates do not change much, but do increase slightly for the composite countries. The most interesting results are the stark changes in exports that accompany relatively small changes in real income. For example, Chinese exports as a share of their own GDP declines slightly, as China produces more of its own consumption, but grows 50% as a share of global GDP. On the other hand, US exports decline by nearly 30% relative to global GDP (and about the same as a share of US GDP). These changes in export activity are also met with changes in trade imbalances across steady states. Table 6 displays the initial and final trade deficits or surpluses run by each country. Changes in the net exports motivates turning to our analysis of dynamics, where we will also see that short run changes in the variables described above are substantially larger than their long run counterparts.

Turning to export dynamics, Figure 5 plots changes in both gross and net exports across coun-

Table 6: Changes in Trade Balances

Country	Initial Imbalances (% World GDP)	Final Imbalances (% World GDP)
China	0.14	0.22
EU	0.59	1.21
UK	0.01	-.05
ROW (L)	-0.22	0.11
ROW (H)	0.73	1.03
USA	-1.26	-2.52

tries. In the short run, export activity surges in the United States, and the the US runs a temporary trade surplus. After the initial surge, exports decline and the US runs a permanent trade deficit. China and the EU also see an increase in exports, but a much smaller amount. On the other hand, in the short run, the ROW composites see large declines in exports and on net deteriorations in the trade balance. In the model, the trade shock occurs in one period and is permanent. The only source of dynamics in output is the misallocation of labor that follows the change in Chinese productivity, followed by reallocation towards a new optimum. Indeed, since there is no capital in our model, the absence of a frictional labor market would lead to immediate reorganization of the economy. On the other hand, initial wealth allocations and the net present value of income determine the amount of consumption that households will demand in both initial and final steady states. Through the Euler Equation, (43), the path of consumption determines how workers (as well as firms, and new entrants) trade off wages today for the future. [1] And so, depending on the marginal utility of additional expenditure, countries facing the same adjustment costs can actually adjust to shocks very differently.

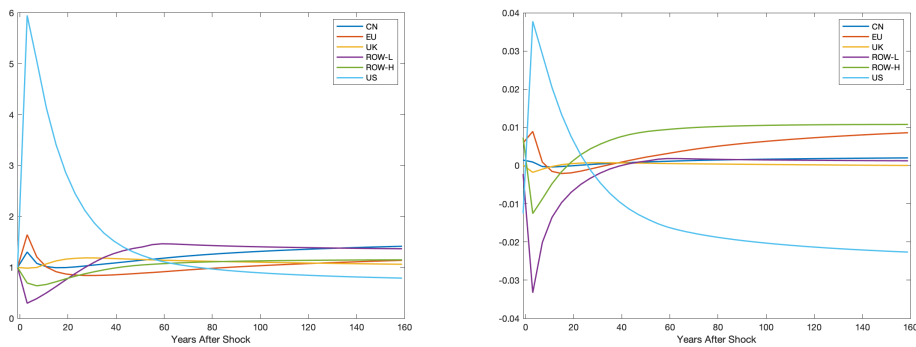
This mechanism has been heretofore absent from models of trade and labor market dynamics. Moreover, the mechanism has important quantitative implications. For example, one implication of (43) and the fact that we assume log utility is that nominal expenditure jumps to its final steady state level immediately. Hence, for example, Chinese nominal expenditure immediately jump to its final steady state level—which is large, as doubling Chinese productivity makes them much richer. However, since reallocation is slow, reaching steady state levels of production takes time. Hence, to finance consumption, China runs a deficit (actually lowering their surplus) in the short run. That is to say, the increase in Chinese consumption increases short run demand for goods produced abroad. Similarly, the ROW composite of poor countries faces immediate import competition from

[1] Formally, with log utility, the Lagrange multiplier on the budget constraint is simply the inverse of nominal per capita consumption.

China. While this will ultimately leave them worse off in the long run, they can adjust somewhat over time. And so, in the short run they too run a deficit. The combined short run demand for imports are such that US export activity actually temporarily surges.

Figure 5: Trade Flows Evolution

(a) Exports   (b) Net Exports



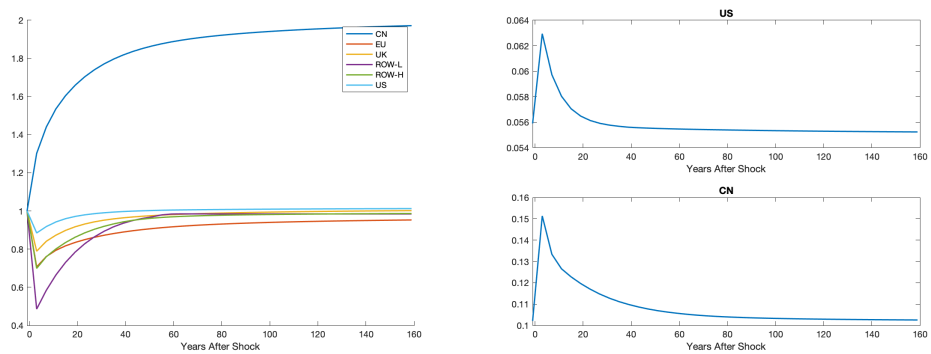
Notes: Plots Q4 transitions for each period. Exports are in terms of GDP share and relative to base year. Net exports are in terms of shares of world GDP and unnormalized.

The changes in export dynamics mirror significant short run differences in consumption dynamics and unemployment. Figure 6 plots the evolution of real expenditure across countries over time. Unsurprisingly, the Chinese economy grows over this time period. On the other hand, all other countries see a decline in *real* consumption in the short run. These declines in real expenditure show that the long run impacts are deceptively small and imprecise measures of the costs of reallocation. Besides employment, figure 6b shows the change in unemployment for specifically the United States and China. In both countries, unemployment increases. This is because reallocation necessarily requires workers to be dislocated from their jobs. Dislocation is partially endogenous, and one could imagine rapid adjustment if many workers are suddenly willing to break their matches and switch sectors. However, two forces in the model temper the rate that workers willingly break matches: first, as workers break matches, labor markets become less tight, and the probability of finding a new job declines; second, switching costs, which are partially stochastic, adds additional risk to workers who break their match. Tying this discussion together, our results here speak to the reduced form literature suggesting there could be both disemployment and income responses to pressure from import competition. However, our model is able to speak to the full path of transition dynamics, and in particular the fact that full adjustment can take many years.

Before concluding, we briefly comment on the actual allocation of workers and wage dynamics.

Figure 6: Real Expenditure & Unemployment

(a) Real Expenditure (b) Unemployment



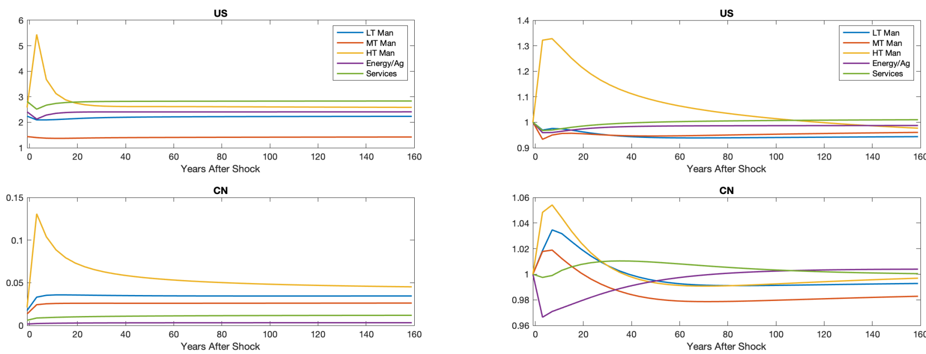
Notes: Plots Q4 transitions for each period. Real expenditure is plotted relative to initial steady state. Unemployment rates are not normalized.

In our model, the risk sharing *within* countries implies that there is no welfare inequality across workers. Nevertheless, the dynamics of prices are a useful benchmark to gauge the order of magnitude of changes in inequality present in models with endogenous imbalances. Figure 7 below plots real wage dynamics across sectors. The fanning out of wages reflects an increase in inequality, while any level shift reflects broad-based income growth.

This figure focuses on comparing the United States and China. In the short run, the United States expands high tech manufacturing. In the long run, services increases as a fraction of the labor force, while high tech manufacturing and agriculture actually return close to their baseline levels. On the other hand, both middle and low tech manufacturing decline in the short and long run. These dynamics play out in wages: high tech manufacturing workers receive a large raise in the short run (this is also the US's export good). As workers reallocate, wages are slowly bid down. In the long run, inequality is close to unchanged. This makes sense because workers are homogenous in the long run, and wage differentials are primarily sustained by differences in moving costs across sectors (which are present in the initial steady state). However, in the short run and medium run labor market frictions induce wild swings in inequality. For example, the real income of high tech manufacturing workers *doubles* in the short run, while it actually declines by nearly 10% in almost all other sectors. On the other hand, all that determined which workers were in which sectors was completely stochastic. This is even more extreme in China, where wages in the high tech manufacturing sector increase *six-fold* but only increase two-fold or less elsewhere. This occurs despite substantially more modest reallocation in China.

Figure 7: Wages & Allocations

(a) Real Wages | (b) Labor Allocation



Notes: Plots Q4 transitions for each period. Real wages defined as nominal sectoral prices, $\tilde{w}^t_{k,i}$, deflated by country-specific price indices. Labor allocations condition on employment and are relative to initial steady state.

6 Going Forward

The productivity shock in China is useful for illustrating the model's mechanics, but is not necessarily a quantitatively useful shock. Nevertheless, to further elucidate our contribution we plan to compare these results to a model without one period bonds, and so the family must consume national income in each period. Going forward, we will always compare these two environments. However, we also plan to analyze three situations: (1) uniform reductions in trade costs; (2) a calibrated version of the model that matches Chinese import growth (the 'China Shock'); and (3) calibrated version of the model that matches changes in Chinese and US trade imbalances (the 'savings glut'). The model we presented in Section 2 will be extended in several dimensions. As soon as we have the basic formulation running, we will extend our framework to allow for (a) inter-sectoral linkages as in Caliendo and Parro (2015); (b) two types of workers (skilled and unskilled); (c) a labor supply decision (that is, a decision to not participate in the labor force); and (d) regions within a country. All of these extensions are conceptually straightforward, and, we hope, will lead to a substantial contribution to our understanding of labor market dynamics in response to trade shocks. A more involved extension we hope to explore is including human capital dynamics into the model. As argued in both Dix-Carneiro (2014) and Traiberman (2019), sector specific experience matters a great deal in determining which workers reallocate and the impact on wages. Adding in sector specific tenure that is destroyed upon switching is straightforward, albeit it increases the state space of the model. The constraint in this setting is data: estimating tenure requires panel data, such as the NLSY or PSID. In this extension, there will likely be a tradeoff on how many margins of adjustment we include. Moreover, this data may be hard to come by for other countries. Nevertheless, we consider the dynamic evolution of human capital a key extension.

7 Conclusion

Appendix

A Household Problem

The Lagrangian of problem (7), (8) and (6) is

$$\mathcal{L} = E_0 \left\{ \begin{array}{c} \sum_{t=0}^{\infty} \left[\delta^t \overline{L} u\left(c_\ell^t\right) - \widetilde{\lambda}^t \left(\overline{L} P^t c_\ell^t + B^{t+1} - \Pi^t - R^t B^t \right) \right] + \\ \int_0^{\overline{L}} \left[\sum_{t=0}^{\infty} \left[\delta^t \left(\begin{array}{c} \left(1 - e_\ell^t\right) \left(-C_{k_\ell^t, k_\ell^{t+1}} + \nu_{\ell, k_\ell^{t+1}}^t + b_{k_\ell^{t+1}} \right) + \\ e_\ell^t \eta_{k_\ell^t} + \widetilde{\lambda}^t \left(\sum_{k=1}^{K} \mathcal{I}\left(k_\ell^{t+1} = k\right) e_\ell^t w_k^t \left(x_\ell^t\right) \right) \end{array} \right) \right] \right] d\ell \end{array} \right\} \tag{A.1}$$

Because each worker is infinitesimal, and the allocation of one worker does not interfere with the allocation/utility of other individual workers (conditional on aggregates), maximizing

$$\int_0^{\overline{L}} \left[\sum_{t=0}^{\infty} \delta^t \left(\begin{array}{c} \left(1 - e_\ell^t\right) \left(-C_{k_\ell^t, k_\ell^{t+1}} + \nu_{\ell, k_\ell^{t+1}}^t + b_{k_\ell^{t+1}} \right) + \\ e_\ell^t \eta_{k_\ell^t} + \widetilde{\lambda}^t \left(\sum_{k=1}^{K} \mathcal{I}\left(k_\ell^{t+1} = k\right) e_\ell^t w_k^t \left(x_\ell^t\right) \right) \end{array} \right) \right] d\ell \tag{A.2}$$

means maximizing each individual term. Therefore, the planner solves, for each individual, the recursive problem:

$$\mathcal{L}_W^t \left(k_\ell^t, e_\ell^t, x_\ell^t, \nu_\ell^t\right) = \max_{k_\ell^{t+1}, \widetilde{e}_k^{t+1}(.)} \left\{ \begin{array}{c} \left(1 - e_\ell^t\right) \left(-C_{k_\ell^t, k_\ell^{t+1}} + \nu_{\ell, k_\ell^{t+1}}^t + b_{k_\ell^{t+1}} \right) + e_\ell^t \eta_{k_\ell^t} \\ + \widetilde{\lambda}_t \sum_{k=1}^{K} \mathcal{I}\left(k_\ell^t = k\right) e_\ell^t w_k^t \left(x_\ell^t\right) + \\ \delta E_t \mathcal{L}_W^{t+1} \left(k_\ell^{t+1}, e_\ell^{t+1}, x_\ell^{t+1}, \nu_\ell^{t+1}\right) \end{array} \right\}. \tag{A.3}$$

Denote by $\mathcal{F}^t$ the set of information at t. So, $E_t(.) = E\left(.|\mathcal{F}^t\right)$. For an unemployed worker in sector k at time t, $k_\ell^t = k$, $e_\ell^t = 0$:

$$\mathcal{L}_W^t \left(k_\ell^t = k, e_\ell^t = 0, x_\ell^t, \nu_\ell^t\right) = \max_{k', \left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'} + \nu_{\ell, k'}^t + b_{k'} + \delta E_t \mathcal{L}_W^{t+1} \left(k', e_\ell^{t+1}, x_\ell^{t+1}, \nu_\ell^{t+1}\right). \tag{A.4}$$

Using the law of iterated expectations we obtain:

$$\begin{aligned}
&\mathcal{L}_W^t\left(k_\ell^t=k,e_\ell^t=0,x_\ell^t,\nu_\ell^t\right)=\max_{k',\left\{\widetilde{e}_k^{t+1}(.)\right\}}-C_{kk'}+\nu_{\ell,k'}^t+b_{k'}\\
&+\delta E\left\{E\left[\mathcal{L}_W^{t+1}\left(k',1,x_\ell^{t+1},\nu_\ell^{t+1}\right)|x_\ell^{t+1},\mathcal{F}^t\right]\times\Pr\left(k_\ell^{t+1}=k',e_\ell^{t+1}=1|x_\ell^{t+1},\mathcal{F}^t\right)|\mathcal{F}^t\right\}\\
&+\delta E\left\{E\left[\mathcal{L}_W^{t+1}\left(k',0,x_\ell^{t+1},\nu_\ell^{t+1}\right)|x_\ell^{t+1},\mathcal{F}^t\right]\times\Pr\left(k_\ell^{t+1}=k',e_\ell^{t+1}=0|x_\ell^{t+1},\mathcal{F}^t\right)|\mathcal{F}^t\right\}\\
&=\max_{k',\left\{\widetilde{e}_k^{t+1}(.)\right\}}-C_{kk'}+\nu_{\ell,k'}^t+b_{k'}\\
&+\delta\left(1-\chi_{k'}\right)\theta_{k'}^t q\left(\theta_{k'}^t\right)E\left\{\mathcal{L}_{t+1}^W\left(k',1,x_{\ell'}^{t+1},\nu_\ell^{t+1}\right)\widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right)|\mathcal{F}^t\right\}\\
&+\delta E\left\{\left(1-\theta_k^t q\left(\theta_{k'}^t\right)\left(1-\chi_{k'}\right)\widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right)\right)\mathcal{L}_W^{t+1}\left(s',0,x_\ell^{t+1},\nu_\ell^{t+1}\right)|\mathcal{F}^t\right\}
\end{aligned}\tag{A.5}$$

For an employed worker in sector k, $k_\ell^t=k$, $e_\ell^t=1$:

$$\begin{aligned}
\mathcal{L}_W^t\left(k_\ell^t=k,e_\ell^t=1,x_\ell^t,\nu_\ell^t\right)&=\max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}}\widetilde{\lambda}^t w_k^t\left(x_\ell^t\right)+\eta_k+\delta E_t\mathcal{L}_W^{t+1}\left(k,e_\ell^{t+1},x_\ell^t,\nu_\ell^{t+1}\right)\\
&=\max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}}\widetilde{\lambda}^t w_k^t\left(x_\ell^t\right)+\eta_k\\
&+\delta E\left\{\begin{array}{c}E\left[\mathcal{L}_W^{t+1}\left(k,1,x_\ell^t,\nu_\ell^{t+1}\right)|k_\ell^{t+1}=k,,e_\ell^{t+1}=1,x_\ell^{t+1},\mathcal{F}^t\right]\times\\ \Pr\left(k_\ell^{t+1}=k,e_\ell^{t+1}=1|x_\ell^{t+1},\mathcal{F}^t\right)|\mathcal{F}^t\end{array}\right\}\\
&+\delta E\left\{\begin{array}{c}E\left[\mathcal{L}_W^{t+1}\left(k,0,x_\ell^t,\nu_\ell^{t+1}\right)|k_\ell^{t+1}=k,e_\ell^{t+1}=0,x_\ell^{t+1},\mathcal{F}^t\right]\times\\ \Pr\left(k_\ell^{t+1}=k,e_\ell^{t+1}=0|x_\ell^{t+1},\mathcal{F}^t\right)|\mathcal{F}^t\end{array}\right\}\\
&=\max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}}\widetilde{\lambda}^t w_k^t\left(x_\ell^t\right)+\eta_k\\
&+\delta\left(1-\chi_k\right)E\left[\begin{array}{c}\widetilde{e}_k^{t+1}\left(x_\ell^t\right)\mathcal{L}_W^{t+1}\left(k,1,x_\ell^t,\nu_\ell^{t+1}\right)\\ +\left(1-\widetilde{e}_k^{t+1}\left(x_\ell^t\right)\right)\mathcal{L}_W^{t+1}\left(k,0,x_\ell^t,\nu_\ell^{t+1}\right)|\mathcal{F}^t\end{array}\right]\\
&+\delta\chi_k E\left[\mathcal{L}_W^{t+1}\left(k,0,x_\ell^t,\nu_\ell^{t+1}\right)|\mathcal{F}^t\right]
\end{aligned}\tag{A.6}$$

Make the following definitions

$$\widetilde{U}_k^t\left(\nu_\ell^t\right)\equiv\mathcal{L}_W^t\left(k_\ell^t=k,e_\ell^t=0,x_\ell^t,\nu_\ell^t\right),\text{ and}$$

$$W_k^t(x)\equiv\mathcal{L}_W^t\left(k_\ell^t=k,e_\ell^t=1,x,\nu_\ell^t\right).\tag{A.7}$$

$\widetilde{U}_k^t\left(\nu_\ell^t\right)$ is the value of unemployment in sector k, conditional on the preference shocks ν_ℓ^t, and $W_k^t(x)$ is the value of a job with match productivity x. Note that $\mathcal{L}_W^t\left(k_\ell^t=k,e_\ell^t=0,x_\ell^t,\nu_\ell^t\right)$ does not depend on x_ℓ^t and $\mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 1, x, \nu_\ell^t\right)$ does not depend on ν_ℓ^t. Rewrite $\widetilde{U}_k^t\left(\nu_\ell^t\right)$ as

$$\begin{aligned}\widetilde{U}_k^t\left(\nu_\ell^t\right) = &\max_{k', \left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'} + \nu_{\ell,k'}^t + b_{k'} \\ &+ \delta\left(1-\chi_{k'}\right)\theta_{k'}^t q\left(\theta_{k'}^t\right)\int W_{k'}^{t+1}(x)\,\widetilde{e}_{k'}^{t+1}(x)\,dG_{k'}(x) \\ &+ \delta\left(1-\theta_{k'}^t q\left(\theta_{k'}^t\right)\left(1-\chi_{k'}\right)\Pr\left(\widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right)=1\right)\right)E_\nu\left(\widetilde{U}_{k'}^{t+1}\left(\nu_\ell^{t+1}\right)\right), \end{aligned} \tag{A.8}$$

and so:

$$\begin{aligned}\widetilde{U}_k^t\left(\nu_\ell^t\right) = &\max_{k', \left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'} + \nu_{\ell,k'}^t + b_{k'} \\ &+ \delta\left(1-\chi_{k'}\right)\theta_{k'}^t q\left(\theta_{k'}^t\right)\int \left(\begin{array}{c} W_{k'}^{t+1}(x)\,\widetilde{e}_{k'}^{t+1}(x) + \\ E_\nu\left(\widetilde{U}_{k'}^{t+1}\left(\nu_\ell^{t+1}\right)\right)\left(1-\widetilde{e}_{k'}^{t+1}(x)\right)\end{array}\right) dG_{k'}(x) \\ &+ \delta\left(1-\left(1-\chi_{k'}\right)\theta_{k'}^t q\left(\theta_{k'}^t\right)\right)E_\nu\left(\widetilde{U}_{k'}^{t+1}\left(\nu_\ell^{t+1}\right)\right). \end{aligned} \tag{A.9}$$

Now, wewrite $W_k^t(x)$ as:

$$\begin{aligned}W_k^t(x) = &\max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t(x) + \eta_k \\ &+ \delta\left(1-\chi_k\right)\widetilde{e}_k^{t+1}(x)\,W_k^{t+1}(x) \\ &+ \delta\left(1-\left(1-\chi_k\right)\widetilde{e}_k^{t+1}(x)\right)E\left(\widetilde{U}_k^{t+1}\left(\nu_\ell^{t+1}\right)\right), \end{aligned} \tag{A.10}$$

and so

$$\begin{aligned}W_k^t(x) = &\max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right) + \eta_k \\ &+ \delta\left(1-\chi_k\right)\left(\widetilde{e}_k^{t+1}(x)\,W_k^{t+1}(x) + \left(1-\widetilde{e}_k^{t+1}(x)\right)E_\nu\left(\widetilde{U}_k^{t+1}\left(\nu_\ell^{t+1}\right)\right)\right) \\ &+ \delta\chi_k E_\nu\left(\widetilde{U}_k^{t+1}\left(\nu_\ell^{t+1}\right)\right). \end{aligned} \tag{A.11}$$

It is now clear that the optimal policy $\widetilde{e}_k^{t+1}(.)$ is:

$$\widetilde{e}_k^{t+1}(x) = \left\{\begin{array}{c} 1 \text{ if } W_k^{t+1}(x) > E_\nu\left(\widetilde{U}_k^{t+1}\left(\nu_\ell^{t+1}\right)\right) \\ 0 \text{ otherwise}\end{array}\right\}. \tag{A.12}$$

Define $U_k^t \equiv E_\nu\left(\widetilde{U}_k^t\left(\nu_\ell^t\right)\right)$. We therefore have the following Bellman equations:

$$U_k^t = E_\nu \left(\begin{array}{c} \max_{k'} - C_{kk'} + b_{k'} + \nu_{\ell,k'}^t \\ +\delta\left(1-\chi_{k'}\right)\theta_{k'}^t q\left(\theta_{k'}^t\right)\int \max\left\{W_{k'}^{t+1}(x), U_{k'}^{t+1}\right\} dG_{k'}(x) \\ +\delta\left(1-\left(1-\chi_{k'}\right)\theta_{k'}^t q\left(\theta_{k'}^t\right)\right)U_{k'}^{t+1} \end{array} \right) \tag{A.13}$$

$$W_k^t(x) = \widetilde{\lambda}^t w_k^t(x) + \eta_k + \delta\left(1-\chi_k\right)\left(\max\left\{W_k^{t+1}(x), U_k^{t+1}\right\}\right) + \delta\chi_k U_k^{t+1} \tag{A.14}$$

B Steady State Equilibrium

In this section we derive the equations characterizing the steady state equilibrium. The key conditions that we impose is that variables are constant over time, inflows of workers into each sector equal outflos, and job destruction rates equal job creation rates.

Wage Equation

$$w_{k,i}(x) = \beta_{k,i}\widetilde{w}_{k,i}x + \frac{\left(1-\beta_{k,i}\right)(1-\delta)U_{k,i} - \left(1-\beta_{k,i}\right)\eta_{k,i}}{\widetilde{\lambda}_i} \tag{A.15}$$

Firms' value function

$$J_{k,i}(x) = \frac{1-\beta_{k,i}}{1-\left(1-\chi_{k,i}\right)\delta}\widetilde{\lambda}_i\widetilde{w}_{k,i}\left(x-\underline{x}_{k,i}\right) \tag{A.16}$$

Probability of filling a vacancy

$$q_i\left(\theta_{k,i}\right) = \frac{\kappa_{k,i}\left(1-\left(1-\chi_{k,i}\right)\delta\right)}{\delta\left(1-\chi_{k,i}\right)\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}_{k,i}\right)}, \tag{A.17}$$

where

$$I_{k,i}\left(\underline{x}_{k,i}\right) \equiv \int_{\underline{x}_{k,i}}^{x_{\max}}\left(s-\underline{x}_{k,i}\right)dG_{k,i}(s) \tag{A.18}$$

Unemployed workers' Bellman equation

$$U_{k,i} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}\kappa_{k',i}\widetilde{\lambda}_i\widetilde{w}_{k',i}\frac{\beta_{k',i}}{1-\beta_{k',i}} + \delta U_{k',i}}{\zeta_i}\right\}\right) \tag{A.19}$$

Transition rates

$$s_{k\ell,i} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}\kappa_{\ell,i}\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{1-\beta_{\ell,i}}+\delta U_{\ell,i}}{\zeta_i}\right\}}{\sum\limits_{\overline{k}}\exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta_{\overline{k},i}\kappa_{\overline{k},i}\widetilde{\lambda}_i\widetilde{w}_{\overline{k},i}\frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}}+\delta U_{\overline{k},i}}{\zeta_i}\right\}} \tag{A.20}$$

Steady-state unemployment rates

$$u_{k,i} = \frac{\chi_{k,i}}{\theta_{k,i}q_i\left(\theta_{k,i}\right)\left(1-\chi_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}\right)\right)+\chi_{k,i}} \tag{A.21}$$

Trade shares

$$\pi_{k,oi} = \frac{\left(\frac{A_{k,o}}{d_{k,oi}\widetilde{w}_{k,o}}\right)^{\lambda}}{\Phi_{k,i}}, \tag{A.22}$$

where

$$\Phi_{k,i} = \sum_{o'}\left(\frac{A_{k,o'}}{d_{k,o'i}\widetilde{w}_{k,o'}}\right)^{\lambda}. \tag{A.23}$$

Price indices

$$P_i = \frac{\prod\limits_{k=1}^{K}\left(P_{k,i}\right)^{\mu_{k,i}}}{\prod\limits_{k=1}^{K}\left(\mu_{k,i}\right)^{\mu_{k,i}}} \tag{A.24}$$

where

$$P_{k,i} = \psi\left(\Phi_{k,i}\right)^{-\frac{1}{\lambda}} \tag{A.25}$$

$$\psi = \left[\Gamma\left(\frac{\lambda+1-\sigma}{\lambda}\right)\right]^{\frac{1}{1-\sigma}} \tag{A.26}$$

Zero net flows condition

$$\left(L_i.u_i\right) = \sum_{\ell=1}^{K} s_{\ell k,i}L_{\ell,i}u_{\ell,i} = s_i'\left(L_i.u_i\right) \tag{A.27}$$

Product market clearing

$$Y_{k,o} = \widetilde{w}_{k,o} L_{k,o} \left(1 - u_{k,o}\right) \int_{\underline{x}_{k,o}}^{x_{\max}} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,o}\right)} dG_{k,o}\left(s\right) \tag{A.28}$$

$$E_i = \sum_{k=1}^{K} \widetilde{w}_{k,i} L_{k,i} \left(1 - u_{k,i}\right) \left(\int_{\underline{x}_{k,i}}^{x_{\max}} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} dG_{k,i}\left(s\right) - \frac{u_{k,i}}{1 - u_{k,i}} \theta_{k,i} \kappa_{k,i} \right) - NX_i \tag{A.29}$$

$$E^V_{k,o} = \kappa_{k,o} \widetilde{w}_{k,o} \theta_{k,o} u_{k,o} L_{k,o} \tag{A.30}$$

$$Y_{k,o} = \sum_{i=1}^{N} \pi_{k,oi} \mu_{k,i} E_i + E^V_{k,o} \tag{A.31}$$

Lagrange multipliers

$$\widetilde{\lambda}_i = \frac{\overline{L}_i}{E_i} \tag{A.32}$$

C Solution Methods

C.1 Algorithm to Compute the Steady-State Equilibrium

- Define $I_{k,i}\left(x\right) \equiv \int_x^{x_{\max}} \left(s - x\right) dG_{k,i}\left(s\right)$. Imposing $G_{k,i} \sim \log \mathcal{N}\left(0, \sigma^2_{k,i}\right)$ and a bit of algebra leads to:
- $G_{k,i}\left(x\right) = \Phi\left(\frac{\ln x}{\sigma_{k,i}}\right)$
- $I_{k,i}\left(x\right) = \exp\left(\frac{\sigma^2_{k,i}}{2}\right) \Phi\left(\sigma_{k,i} - \frac{\ln x}{\sigma_{k,i}}\right) - x\Phi\left(-\frac{\ln x}{\sigma_{k,i}}\right)$
- $I_{k,i}\left(0\right) = \exp\left(\frac{\sigma^2_{k,i}}{2}\right)$
- $\int_{\underline{x}_{k,i}}^{x_{\max}} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} dG_{k,i}\left(s\right) = \exp\left(\frac{\sigma^2_{k,i}}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}$

Step 0: Define

$$\varpi_{k,i} \equiv \frac{\left(1 - \left(1 - \chi_{k,i}\right)\delta\right)\kappa_{k,i}}{\delta\left(1 - \beta_{k,i}\right)\left(1 - \chi_{k,i}\right)}$$

If $\frac{\left(1-\left(1-\chi_{k,i}\right)\delta\right)\kappa_{k,i}}{\delta\left(1-\beta_{k,i}\right)\left(1-\chi_{k,i}\right)I_{k,i}(0)} = \frac{\varpi_{k,i}}{I_{k,i}(0)} \geq 1$, the free entry condition cannot be satisfied—$I_{k,i}$ is decreasing. Abort the procedure and highly penalize the objective function.

Step 1: Find $\underline{x}_{k,i}^{ub}$ such that $\frac{\left(1-\left(1-\chi_{k,i}\right)\delta\right)\kappa_{k,i}}{\delta\left(1-\beta_{k,i}\right)\left(1-\chi_{k,i}\right)I_{k,i}\left(\underline{x}_{k,i}^{ub}\right)} = 1 \iff I_{k,i}\left(\underline{x}_{k,i}^{ub}\right) = \varpi_{k,i}$. If along the algorithm $\underline{x}_{k,i}$ goes above $\underline{x}_{k,i}^{ub}$, we update it to be equal to $\underline{x}_{k,i}^{ub}$ (minus a small number).

Step 2: Guess $\{L_{k,i}\}$, and $\left\{\underline{x}_{k,i}\right\}$

Step 3: Compute $I_{k,i}\left(\underline{x}_{k,i}\right)$, $G_{k,i}\left(\underline{x}_{k,i}\right)$, $\theta_{k,i}$ and $u_{k,i}$.

- $\theta_{k,i} = q_i^{-1}\left(\frac{\varpi_{k,i}}{I_{k,i}\left(\underline{x}_{k,i}\right)}\right)$ where $q_i^{-1}(y) = \left(\frac{1-y^{\xi_i}}{y^{\xi_i}}\right)^{1/\xi_i}$
- $u_{k,i} = \frac{\chi_{k,i}}{\theta_{k,i}q_i\left(\theta_{k,i}\right)\left(1-\chi_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}\right)\right)+\chi_{k,i}}$

Step 4: Compute $\widetilde{L}_{k,i}$

$$\begin{aligned}\widetilde{L}_{k,i} &\equiv L_{k,i}\left(1-u_{k,i}\right)\int_{\underline{x}_{k,i}}^{x_{\max}} \frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}\right)} dG_{k,i}(s) \\ &= L_{k,i}\left(1-u_{k,i}\right)\exp\left(\frac{\sigma_{k,i}^2}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}\end{aligned}$$

Step 5: Obtain $\{\widetilde{w}_{k,i}\}$

- Step 5a: Guess $\widetilde{w}_{k,i}$
- Step 5b: Compute price indices $P_i = P_i\left(\{\widetilde{w}_{k,i}\}\right)$
- Step 5c: Compute $Y_{k,i} = \widetilde{w}_{k,i}\widetilde{L}_{k,i}$
- Step 5d: Compute $E_{k,i}^V = \kappa_{k,i}\widetilde{w}_{k,i}\theta_{k,i}u_{k,i}L_{k,i}$
- Step 5e: Compute

$$E_i = \sum_{k=1}^{K}\left(Y_{k,i}-E_{k,i}^V\right) - \left(\sum_{k=1}^{K} Y_{k,US} - E_{k,US}^V\right)\frac{NX_i^{Data}}{GDP_{US}^{Data}}$$

where $\frac{NX_i^{Data}}{GDP_{US}^{Data}}$ is net exports in country o as a fraction of US GDP in the data.

- Step 5f: Compute $\pi_{k,oi} = \frac{\left(A_{k,o}/\left(d_{k,oi}\widetilde{w}_{k,o}\right)\right)^{\lambda}}{\sum_{o'}\left(A_{k,o'}/\left(d_{k,o'i}\widetilde{w}_{k,o'}\right)\right)^{\lambda}}$

- Step 5g: Compute $Dem_{k,i} = \sum\limits_{i=1}^{N} \pi_{k,oi}\mu_{k,i}E_i + E_{k,o}^{V}$

- Step 5h: Update $(\widetilde{w}_{k,i})' = \frac{Dem_{k,i}}{\widetilde{L}_{k,i}}$.

- Step 5i: Go to Step 5b until convergence.

Step 6: Compute Lagrange Multipliers $\widetilde{\lambda}_i = \frac{\overline{L}_i}{E_i}$

Step 7: Obtain $\{U_{k,i}\}$.

- Step 7a: Guess $\left\{U_{ki}^{0}\right\}$

- Step 7b: Compute until convergence

$$U_{k,i}^{g+1} = \zeta_i \log\left(\sum_{\ell=1}^{K} \exp\left\{\frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}\kappa_{\ell,i}\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{1-\beta_{\ell,i}} + \delta U_{\ell,i}^{g} - \delta U_{k,i}^{g}}{\zeta_i}\right\}\right) + \delta U_{k,i}^{g}$$

Step 8: Update $L_{k,i}$.

- Step 8a: Given knowledge of $\{U_{k,i}\}$, compute transition rates $s_{k\ell,i}$.

$$s_{k\ell,i} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}\kappa_{\ell,i}\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{1-\beta_{\ell,i}}+\delta U_{\ell,i}}{\zeta_i}\right\}}{\sum\limits_{\overline{k}} \exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta_{\overline{k},i}\kappa_{\overline{k},i}\widetilde{\lambda}_i\widetilde{w}_{\overline{k},i}\frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}}+\delta U_{\overline{k},i}}{\zeta_i}\right\}}$$

- Step 8b: Find y_i such that

$$\left(I - s_i'\right) y_i = 0$$

- Step 8c: Find allocations $L_{k,i}$

$$L_{k,i}u_{k,i} = \varphi y_{k,i}$$

$$
\begin{aligned}
&\Rightarrow L_{k,i} = \varphi \underbrace{y_{k,i}/u_{k,i}}_{\widetilde{y}_{k,i}} \\
&\Rightarrow L_i' 1_{K\times 1} = \varphi \widetilde{y}_{k,i}' 1_{K\times 1} = \overline{L}_i \\
&\Rightarrow \varphi = \frac{\overline{L}_i}{\widetilde{y}_{k,i}' 1_{K\times 1}}
\end{aligned}
$$

$$(L_{k,i})' = \varphi \widetilde{y}_{k,i}$$

$$L_{k,i}^{new} = (1-\lambda_L) L_{k,i} + \lambda_L (L_{k,i})'$$

Step 9: Update $\underline{x}_{k,i}$.

Note that, in equilibrium:

$$\widetilde{\lambda}_i \widetilde{w}_{k,i} \underline{x}_{k,i} = (1-\delta) U_{k,i} - \eta_{k,i}$$

So, we update $\underline{x}_{k,i}$ according to:

$$(\underline{x}_{k,i})' = \frac{(1-\delta) U_{k,i} - \eta_{k,i}}{\widetilde{\lambda}_i \widetilde{w}_{k,i}}$$

$$\underline{x}_{k,i}^{new} = \min \left\{ (1-\lambda_x) \underline{x}_{k,i} + \lambda_x (\underline{x}_{k,i})', \underline{x}_{k,i}^{ub} \right\}$$

Step 10: Armed with $L_{k,i}^{new}$ and $\underline{x}_{k,i}^{new}$ go to Step 3 until $\left\| \left\{ L_{k,i}^{new} - L_{k,i} \right\} \right\| \to 0$ and $\left\| \left\{ \underline{x}_{k,i}^{new} - \underline{x}_{k,i} \right\} \right\| \to 0$.

Note that $\left\| \left\{ \underline{x}_{k,i}^{new} - \underline{x}_{k,i} \right\} \right\| \to 0$ does not imply that (**??**) is satisfied. Therefore, we penalize deviations from (**??**) in the objective function.

C.2 Expressions for Simulated Moments

C.2.1 Employment Shares

$$emp_{k,i} = \frac{L_{k,i} (1-u_{k,i})}{\sum_{k=1}^{K} L_{k,i} (1-u_{k,i})}$$

C.2.2 National Unemployment Rate

$$unemp_i = \frac{\sum_{k=1}^{K} L_{k,i} u_{k,i}}{\sum_{k=1}^{K} L_{k,i}}$$

C.2.3 Sector-Specific Average Wages

$$w_{k,i}(x) = (1 - \beta_{k,i}) \widetilde{w}_{k,i} \underline{x}_{k,i} + \beta_{k,i} \widetilde{w}_{k,i} x$$

$$\begin{aligned}
\overline{w}_{k,i} &= \frac{\int_{\underline{x}_{k,i}}^{x_{\max}} w_{k,i}(s)\, dG_{k,i}(s)}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} \\
&= (1 - \beta_{k,i}) \widetilde{w}_{k,i} \underline{x}_{k,i} + \beta_{k,i} \widetilde{w}_{k,i} \int_{\underline{x}_{k,i}}^{x_{\max}} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} dG_{k,i}(s) \\
&= (1 - \beta_{k,i}) \widetilde{w}_{k,i} \underline{x}_{k,i} + \beta_{k,i} \widetilde{w}_{k,i} \exp\left(\frac{\sigma_{k,i}^2}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}
\end{aligned}$$

C.2.4 Sector-Specific Variance of Wages

$$\begin{aligned}
\sigma_{w,k,i}^2 &= \frac{\int_{\underline{x}_{k,i}}^{\infty} \left(w_{k,i}(s) - \overline{w}_{k,i}\right)^2 dG_{k,i}(s)}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} \\
&= \left(\beta_{k,i} \widetilde{w}_{k,i}\right)^2 \times \frac{\int_{\underline{x}_{k,i}}^{\infty} \left(s - \exp\left(\frac{\sigma_{k,i}^2}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}\right)^2 dG_{k,i}(s)}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} \\
&= \left(\beta_{k,i} \widetilde{w}_{k,i}\right)^2 \times \left(\exp\left(2\sigma_{k,i}^2\right) \frac{\Phi\left(2\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)} - \exp\left(\sigma_{k,i}^2\right) \left(\frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}\right)^2\right)
\end{aligned}$$

C.2.5 Transition Rates

Note that the transition rates in equation (12) are transitions from unemployment in sector k to search in sector k' within period t. There are no data counterfactuals for this variable. However, we can construct a matrix with transition rates between all possible (model) states between time t and time $t + N$ (where N is even)—where variables are measured at the t_a stage (which is the production stage). From this matrix, we can obtain N-period transition rates between all states observed in the data (employment in each of the sectors and unconditional unemployment). First, we obtain the one-year transition matrix $\widetilde{s}^{t,t+1}$ between states $\{\widetilde{u}_1, ..., \widetilde{u}_K, 1, ..., K\}$. Here, we abuse notation to mean $\widetilde{u}_k$ as sector-k unemployment at the very beginning of a period.

The one-year transition rate between sector-ℓ unemployment and sector-k unemployment is given by:

$$\widetilde{s}^{t,t+1}_{\widetilde{u}_\ell \widetilde{u}_k,i} = s^t_{\ell k,i} \left(1 - \theta^t_{k,i} q_i \left(\theta^t_{k,i}\right) (1 - \chi_{k,i}) \left(1 - G_{k,i}\left(\underline{x}^{t+1}_{k,i}\right)\right)\right), \tag{A.33}$$

that is, a share $s^t_{\ell k,i}$ of individuals starting period t unemployed in sector ℓ choose to search in sector k. A fraction $\left(1 - \theta^t_{k,i} q_i \left(\theta^t_{k,i}\right) (1 - \chi_{k,i}) \left(1 - G_{k,i}\left(\underline{x}^{t+1}_{k,i}\right)\right)\right)$ of those do not find a match that survives until $t+1$. Similarly, the one-year transition rate between sector-ℓ unemployment and sector-k employment is given by:

$$\begin{aligned}\widetilde{s}^{t,t+1}_{\widetilde{u}_\ell k,i} &= s^t_{\ell k,i} \theta^t_{k,i} q_i \left(\theta^t_{k,i}\right) (1 - \chi_{k,i}) \left(1 - G_{k,i}\left(\underline{x}^{t+1}_{k,i}\right)\right) \\ &= s^t_{\ell k,i} - \widetilde{s}^{t,t+1}_{\widetilde{u}_\ell \widetilde{u}_k,i}.\end{aligned} \tag{A.34}$$

According to the timing assumptions of the model, the one-year transition rate between employment in sector k and employment in sector k' is zero if $k \neq k'$. However, the persistence rate of employment in sector k is given by the probability that a match does not receive a death shock times the probability that the match is not dissolved because the threshold for production increases in the following period:

$$\widetilde{s}^{t,t+1}_{kk',i} = \begin{cases} 0 \text{ if } k \neq k' \\ (1 - \chi_{k,i}) \Pr\left(x \geq \underline{x}^{t+1}_{k,i} | x \geq \underline{x}^t_{k,i}\right) \text{ if } k = k' \end{cases}. \tag{A.35}$$

Finally, the one-year transition rate between sector-k employment and unemployment in sector ℓ is given by:

$$\widetilde{s}^{t,t+1}_{k\widetilde{u}_\ell,i} = \begin{cases} 0 \text{ if } k \neq \ell \\ \chi_{k,i} + (1 - \chi_{k,i}) \Pr\left(x < \underline{x}^{t+1}_{k,i} | x \geq \underline{x}^t_{k,i}\right) \text{ if } k = \ell \end{cases}. \tag{A.36}$$

That is, if a worker is employed in sector k at t, she cannot start next period unemployed in sector ℓ if $k \neq \ell$. Otherwise, workers transition between sector k employment to sector k unemployment if their match is hit with a death shock or if their employer's productivity goes below the threshold for production at $t+1$.

We can now write the N-period transition matrix as:

$$\widetilde{s}^{t,t+N} = \widetilde{s}^{t+k-1,t+k} \times ... \times \widetilde{s}^{t+1,t+2} \times \widetilde{s}^{t,t+1}, \tag{A.37}$$

and we can write transition rates between unemployment $\widetilde{u}$ and sector-k employment between t and $t+N$ as:

$$\widetilde{s}_{\widetilde{u},k,i}^{t,t+N}=\frac{\sum_{\ell=1}^{K}L_{\ell,i}^{t-1}\widetilde{u}_{\ell,i}^{t-1}\widetilde{s}_{\widetilde{u}_\ell,k}^{t,t+N}}{\sum_{\ell=1}^{K}L_{\ell,i}^{t-1}\widetilde{u}_{\ell,i}^{t-1}}. \tag{A.38}$$

Finally, we can write transition rates between sector-k employment and unemployment $\widetilde{u}$ as:

$$\widetilde{s}_{k,\widetilde{u},i}^{t,t+N}=1-\sum_{k'=1}^{K}\widetilde{s}_{k,k',i}^{t,t+N}. \tag{A.39}$$

1-period transition rates

$$\widetilde{s}_{\widetilde{u}_\ell\widetilde{u}_k,i}=s_{\ell k,i}\left(1-\theta_{k,i}q_i\left(\theta_{k,i}\right)\left(1-\chi_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}\right)\right)\right)$$

$$\widetilde{s}_{\widetilde{u}_\ell k,i}=s_{\ell k,i}\theta_{k,i}q_i\left(\theta_{k,i}\right)\left(1-\chi_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}\right)\right)$$

$$\widetilde{s}_{\ell k,i}=\begin{cases}0 \text{ if } \ell\neq k\\ \left(1-\chi_{\ell,i}\right) \text{ if } \ell=k\end{cases}$$

$$\widetilde{s}_{\ell\widetilde{u}_k,i}=\begin{cases}0 \text{ if } \ell\neq k\\ \chi_{k,i} \text{ if } \ell=k\end{cases}$$

N-period transition rates from and to unconditional unemployment: $\widetilde{s}^N$

$$\widetilde{s}_{\widetilde{u},k,i}^{N}=\frac{\sum_{\ell=1}^{K}L_{\ell,i}u_{\ell,i}\widetilde{s}_{\widetilde{u}_\ell,k,i}^{N}}{\sum_{\ell=1}^{K}L_{\ell,i}u_{\ell,i}}$$

$$\widetilde{s}_{k,\widetilde{u},i}^{N}=1-\sum_{\ell=1}^{K}\widetilde{s}_{k,\ell,i}^{N}.$$

C.3 Algorithm: Out-of-Steady-State Transition

Note that $\widetilde{\lambda}_i^t=\frac{u'\left(c_i^t\right)}{P_i^t}=\frac{1}{P_i^tc_i^t}=\frac{\overline{L}_i}{E_i^t}$, where $E_i^t\equiv P_i^tC_i^t=\overline{L}_iP_i^tc_i^t$

Also, note that, since we are normalizing Global GDP to 1, we have

$VA_i^t = \sum_{k=1}^{K}\left(Y_{k,i} - E_{k,i}^V\right)$ and $\sum_{i=1}^{N} VA_i^t = 1$

$R^{t+1} = \frac{1}{\delta}\frac{\sum_{i=1}^{N} VA_i^{t+1}}{\sum_{i=1}^{N} VA_i^t} = \frac{1}{\delta}$

So the Euler equation gives $P_i^{t+1}C_i^{t+1} = \delta R^{t+1} P_i^t C_i^t \Rightarrow P_i^{t+1}C_i^{t+1} = P_i^t C_i^t \Rightarrow E_i^{t+1} = E_i^t = E_i$

Therefore, $\widetilde{\lambda}_i^t = \widetilde{\lambda}_i = \frac{\overline{L}_i}{E_i}$

Inner Loop: conditional on path for expenditures $E_i^t = E_i$

Step 3: Set $\widetilde{\lambda}_i^t = \widetilde{\lambda}_i^{T_{SS}}$

Step 4: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}-1}$ for each sector k and country i.

Step 4a: Compute sequence $\left\{P_i^t\right\}_{t=1}^{T_{SS}-1}$ for each country i

Step 5: Given full knowledge of all equilibrium variables at T_{SS}, start at $t = T_{SS} - 1$ and sequentially compute (backwards) for each $t = T_{SS} - 1, ..., 1$

- Step 5a: Given $\widetilde{w}_{k,i}^t$, P_i^t, $\underline{x}_{k,i}^{t+1}$, δ_i^{t+1} and $J_{k,i}^{t+1}(s)$ compute $\theta_{k,i}^t$

$$\theta_{k,i}^t = q_i^{-1}\left(\frac{\widetilde{\lambda}_i^t \kappa_{k,i}\widetilde{w}_{k,i}^t}{\delta\left(1-\chi_{k,i}\right)\int_{\underline{x}_{k,i}^{t+1}}^{x_{\max}} J_{k,i}^{t+1}(s)\,dG_{k,i}(s)}\right)$$

- Step 5b: Given $\underline{x}_{k,i}^{t+1}$, $W_{k,i}^{t+1}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}} J_{k,i}^{t+1}(x) + U_{k,i}^{t+1}$ (for $x \geq \underline{x}_{k,i}^{t+1}$), $\theta_{k,i}^t$, $U_{k,i}^{t+1}$ compute $U_{k,i}^t$. Notice that $\int_{\underline{x}_{k',i}^{t+1}}^{x_{\max}} W_{k',i}^{t+1}(s)\,dG_{k',i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}}\int_{\underline{x}_{k,i}^{t+1}}^{x_{\max}} J_{k,i}^{t+1}(s)\,dG_{k,i}(s) + \left(1 - G_{k,i}\left(\underline{x}_{k',i}^{t+1}\right)\right)U_{k,i}^{t+1}$ so we do not need to recompute the integral.

$$U_{k,i}^t = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{\begin{array}{c}-C_{kk',i} + b_{k',i}\\ +\delta\theta_{k',i}^t q_i\left(\theta_{k',i}^t\right)\left(1-\chi_{k',i}\right)\int_{\underline{x}_{k',i}^{t+1}}^{x_{\max}} W_{k',i}^{t+1}(s)\,dG_{k',i}(s) + \\ \delta\left(1-\theta_{k',i}^t q_i\left(\theta_{k',i}^t\right)\left(1-\chi_{k',i}\right)\left(1-G_{k,i}\left(\underline{x}_{k',i}^{t+1}\right)\right)\right)U_{k',i}^{t+1}\end{array}}{\zeta_i}\right\}\right)$$

- Step 5c: Given $J_{k,i}^{t+1}(x)$, $\widetilde{w}_{k,i}^t$, $\theta_{k,i}^t$, δ_i^{t+1}, $U_{k,i}^t$, $U_{k,i}^{t+1}$ and $\underline{x}_{k,i}^{t+1}$ compute $J_{k,i}^t(x)$

$$\begin{aligned} J_{k,i}^t(x) = & (1-\beta_{k,i})\widetilde{\lambda}_i^t \widetilde{w}_{k,i}^t x + (1-\beta_{k,i})\eta_{k,i} \\ & -(1-\beta_{k,i})\left(U_{k,i}^t - \delta U_{k,i}^{t+1}\right) + (1-\chi_{k,i})\delta \max\left\{J_{k,i}^{t+1}(x), 0\right\} \end{aligned}$$

- Step 5d: Solve for $\underline{x}_{k,i}^t$: $J_{k,i}^t\left(\underline{x}_{k,i}^t\right) = 0$

Step 6: Compute transition rates $\left\{s_{kk',i}^t\right\}_{t=1}^{T_{SS}-1}$ for all countries i using (12).

Step 7: Compute $\left\{\pi_{k,oi}^t\right\}_{t=1}^{T_{SS}-1}$ for all countries i

$$\pi_{k,oi}^t = \frac{\left(A_{k,o}/\left(d_{k,oi}\widetilde{w}_{k,o}^t\right)\right)^{\lambda}}{\sum_{o'}\left(A_{k,o'}/\left(d_{k,o'i}\widetilde{w}_{k,o'}^t\right)\right)^{\lambda}}$$

Step 8: Start loop over t going forward ($t = 0$ to $t = T_{SS} - 2$)

Initial conditions: we know $\widetilde{u}_{k,i}^{t=-1} = u_{k,i}^{t=0}$, $L_{k,i}^{t=-1} = L_{k,i}^{t=0}$, and $\theta_{k,i}^{t=0}$ from the initial steady state computation. Obtain $\widetilde{u}_{k,i}^t$ and $L_{k,i}^t$ using flow conditions and sequences $\left\{\theta_{k,i}^t\right\}$, $\left\{\underline{x}_{k,i}^t\right\}$.

- Step 8a: Compute

$$JC_{k,i}^t = L_{k,i}^t u_{k,i}^t \theta_{k,i}^t q_i\left(\theta_{k,i}^t\right)(1-\chi_{k,i})\left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)$$

$$JD_{k,i}^t = \left(\chi_{k,i} + (1-\chi_{k,i})\max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^t\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}^t\right)}, 0\right\}\right) L_{k,i}^{t-1}\left(1 - \widetilde{u}_{k,i}^{t-1}\right)$$

$$\widetilde{u}_{k,i}^t = \frac{L_{k,i}^t u_{k,i}^t - JC_{k,i}^t + JD_{k,i}^t}{L_{k,i}^t}$$

- Step 8b: Compute

$$L_{k,i}^{t+1} = L_{k,i}^t + IF_{k,i}^{t+1} - OF_{k,i}^{t+1},$$

where

$$IF_{k,i}^{t+1} = \sum_{\ell \neq k} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1},$$

and

$$OF_{k,i}^{t+1} = L_{k,i}^{t} \widetilde{u}_{k,i}^{t} \left(1 - s_{kk,i}^{t+1}\right).$$

- Step 8c: Compute

$$u_{k,i}^{t+1} = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1}}{L_{k,i}^{t+1}}$$

- Step 8d: Compute

$$\begin{aligned}
\widetilde{L}_{k,i}^{t+1} &= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \int_{\underline{x}_{k,i}^{t+1}}^{\infty} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)} dG_{k,i}\left(s\right) \\
&= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \exp\left(\frac{\sigma_{k,i}^{2}}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}
\end{aligned}$$

$$\text{and } Y_{k,i}^{t+1} = \widetilde{w}_{k,i}^{t+1} \widetilde{L}_{k,i}^{t+1}$$

- Step 8e: Compute

$$E_{k,i}^{V,t+1} = \kappa_{k,i} \widetilde{w}_{k,i}^{t+1} \theta_{k,i}^{t+1} u_{k,i}^{t+1} L_{k,i}^{t+1}$$

- Step 8f: Compute $Dem_{k,i}^{t+1} = \sum_{o=1}^{N} \pi_{k,io}^{t+1} \mu_{k,o} E_{i}^{t+1} + E_{k,i}^{V,t+1}$

- Step 8g: Update $\left(\widetilde{w}_{k,i}^{t+1}\right)' = \frac{Dem_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

- Step 8h: Normalize $\left(\widetilde{w}_{k,i}^{t+1}\right)' = \frac{\left(\widetilde{w}_{k,i}^{t+1}\right)'}{\sum_{i=1}^{N}\sum_{k=1}^{K} VA_{k,i}^{t+1}}$

- Step 8i: Update $\widetilde{w}_{k,i}^{t+1} = \lambda_w \widetilde{w}_{k,i}^{t+1} + (1-\lambda_w)\left(\widetilde{w}_{k,i}^{t+1}\right)'$

Step 9: At this point, we have a new series for $\left\{\widetilde{w}_{k,i}^{t}\right\}$—go back to Step 4a until convergence of $\left\{\widetilde{w}_{k,i}^{t}\right\}$.

Outer Loop: iteration on $\{NX_i^t\}$

Step 0: Impose a change in a subset of parameters that happens at $t=0$, but between t_c and t_d. That is, the shock occurs **after** production, workers' decisions of where to search and after firms post vacancies at $t=0$. Impose a large value for T_{SS}, that is, $T_{SS} \sim 50$ or 100. Assume that for $t \geq T_{SS}$ the system will have converged to a new steady state. World GDP $\sum_{i=1}^{N}\sum_{k=1}^{K} VA_{k,i}^{t}$ is normalized to 1 for every t.

Step 1: Compute steady state equilibrium at $t=0$, conditional on NX_i^0.

Step 2: Compute path $\{R^t\}_{t=1}^{T_{SS}}$ using equation (47), and the normalization $\sum_{i=1}^{N}\sum_{k=1}^{K} VA_{k,i}^{t+1} = 1$:

$$R^{t+1} = \frac{1}{\delta}\frac{\sum_{i=1}^{N}\sum_{k=1}^{K} VA_{k,i}^{t+1}}{\sum_{i=1}^{N}\sum_{k=1}^{K} VA_{k,i}^{t}} = \frac{1}{\delta} = R$$

Step 3: Obtain B_i^0. This will be recovered from the initial steady state value NX_i^0, which is imposed in the estimation procedure to be equal to $\frac{NX_i^{Data}}{GDP_{World}^{Data}}\sum\limits_{i=1}^{N}\sum\limits_{k=1}^{K} VA_{k,i}$, where $VA_{k,i} = Y_{k,i} - E_{k,i}$. We normalize $\sum\limits_{i=1}^{N}\sum\limits_{k=1}^{K} VA_{k,i} = 1$ in estimation, so $NX_i^0 = \frac{NX_i^{Data}}{GDP_{World}^{Data}}$. Using equation (50) we obtain that the steady-state bond returns are given by $R = \frac{1}{\delta}$. Equation (51) then gives us:

$$B_i^0 = \frac{NX_i^0}{\left(1 - \frac{1}{\delta}\right)}$$

Step 4: Notice that we only need to iterate over $NX_i^{T_{SS}}$. Make initial guess for $NX_i^{T_{SS}}$.

Step 5: Compute steady state equilibrium at T_{SS}, conditional on $NX_i^{T_{SS}}$, and the change in parameter values.

Step 6: Obtain path $\{P_i^t C_i^t\}_{t=1}^{T_{SS}}$ using equation (43)—the Euler equation.

$$P_i^{t+1} C_i^{t+1} = P_i^t C_i^t = E_i$$

where E_i is given by the steady state solution at T_{SS}.

Step 7: Solve for the out-of-steady-state dynamics conditional on aggregate expenditures $P_i^t C_i^t = E_i$ for every period.

Step 8: Obtain path $\left\{B_i^t\right\}_{t=1}^{T_{SS}}$ using equation (41), the path for $\left\{R^t\right\}_{t=1}^{T_{SS}}$ and B_i^0 computed in Steps 2 and 3.

$$\begin{aligned} B_i^{t+1} &= \sum_{k=1}^{K} VA_{k,i}^t + R^t B_i^t - P_i^t C_i^t \\ &= \sum_{k=1}^{K} VA_{k,i}^t + \frac{1}{\delta} B_i^t - E_i \end{aligned}$$

Step 9: Obtain path $\left\{NX_i^t\right\}_{t=1}^{T_{SS}}$ using equation (48) and the steady condition:

$$NX_i^t = B_i^{t+1} - R^t B_i^t \text{ for } t < T_{SS}$$

$$\left(NX_i^{T_{SS}}\right)' = \left(1 - \frac{1}{\delta}\right) B_i^{T_{SS}} \text{ for } t = T_{SS}$$

Step 10: Update $NX_i^{T_{SS}}$

$$NX_i^{T_{SS}} = \lambda_o NX_i^{T_{SS}} + (1 - \lambda_0) \left(NX_i^{T_{SS}}\right)'$$

Go back to Step 5 until convergence of $\left\{NX_i^{T_{SS}}\right\}$.

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